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LETTER

A General Perfect Cyclic Interference Alignment by Propagation Delay for Arbitrary X Channels with Two Receivers

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SUMMARY Interference alignment (IA) in temporal domain is important in the case of single-antenna vehicle communications. In this paper, perfect cyclic IA based on propagation delay is extended to the $K \times 2$ X channels with two receivers and arbitrary transmitters $K \geq 2$, which achieves the maximal multiplexing gain by obtaining the theoretical degree of freedom of $2K/(K+1)$. We deduce the alignment and separability conditions, and propose a general scheme which is flexible in setting the index of time-slot for IA at the receiver side. Furthermore, the feasibility of the proposed scheme in the two-/three- Euclidean space is analyzed and demonstrated.

key words: *$K \times 2$ X channel, propagation delay, cyclic interference alignment, degree of freedom, feasibility*

1. Introduction

Interference alignment (IA) technique [1] has become one of the most promising approaches, which overlaps the interference in the same signal space to increase the multiplexing gain. It has been widely applied in many models such as interference channels (IC) [1] and X channels (XC) [2] and helps achieve the maximal multiplexing gain indicated by the concept of degree of freedom (DoF).

Basically IA can be implemented in the spatial domain. There are many works such as [3], [4] on applying IA in multiple-antenna systems such as the 3GPP long-term evolution. On the other hand, if each node is equipped with only single antenna, implementation of IA in temporal domain provides an important alternative at the cost of relatively large propagation delay (PD).

Seeking IA in the temporal domain attracted some attentions such as [5]–[8], which utilizes the PD difference among links and is suitable for the scenario of long PD communications including underwater acoustic networks (UANs). In [9], perfect PD-based cyclic IA was first studied for the 2×2 XC and K-user IC to achieve the DoF of 4/3 and $K/2$, respectively. Recently, we investigate the perfect IA based on proper PD structure for $K \times 2$ XC in [10], while further node placement analysis is given in [11]. Moreover, for the three-user X channel, since perfect IA is impossible [12], we propose a feasible scheme with one extra time slot to achieve the maximum DoF by allowing one more message based on the cyclic IA approach [13].

However, [10] provided only one feasible PD pattern with perfect IA for $K \times 2$ XC and pointed out that the $K = 2$ case is a special case of the cyclic IA [9]. It is natural to ask that if the perfect PD-based cyclic IA can be extended to the general $K \times 2$ XC, which leads us to this article.

In this work, we successfully extend the cyclic IA technique for XC from the basic 2×2 case to the general $K \times 2$ following the framework of the cyclic PD-based model [9]. The alignment and separability conditions of perfect cyclic IA are deduced, which achieves the DoF of $2K/(K+1)$. Compared to [10], a general scheme with $K+1$ PD patterns is provided, where the index of time-slot for perfect cyclic IA at the receiver side can be arbitrarily determined by a system parameter K_1. Furthermore, the feasibility condition is deduced in the Euclidean space.

2. System Model

The $K \times 2$ XC has K ($K \geq 2$) source nodes $\mathcal{S}_k, \forall k \in \{1, 2, \cdots, K\}$ and two destination nodes $\mathcal{D}_j, \forall j \in \{1, 2\}$, where each node has single antenna. The message from $\mathcal{S}_k$ to $\mathcal{D}_j$ is denoted by $W_{k,j}$. Totally, there are $M = 2K$ independent and different messages. Perfect synchronization is assumed among all communication nodes.

Equally sized time-slots are partitioned and normalized for the channel, while each message occupies one time-slot. The transmission is divided into $n \in \mathbb{N}$ consecutive time-slots as a cycle, i.e., the cyclic delays are unrolled over time. The PD between any $\mathcal{S}_k$ and $\mathcal{D}_j$ are assumed to be static and non-negative integer multiples of one time-slot. The channel access is stationary over a period of n consecutive time-slots. Within the stationary period, delayed messages are cyclically right-shifted. After n time-slots new messages are transmitted and the procedure repeats again.

Similar to [9], cyclic right-shift by polynomials in x modulo $x^n - 1$ is used for modeling. Single time-slot in the period of n time-slots is addressed by offsets $x^0, x^1, \cdots, x^{n-1}$, from 0 (no offset) to $n-1$ (maximal offset). Here zero-offset indicates a reference PD. The integer PD between $\mathcal{S}_k$ to $\mathcal{D}_j$ is denoted by $\tau_{k,j} \in \mathbb{D} = \left\{x^i | i = 0, 1, \cdots, n-1\right\}$. We define the following $K \times 2$ PD matrix $\boldsymbol{D}$ with elements $d_{k,j}$ in polynomial form

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$$\boldsymbol{D} = \begin{bmatrix} d_{1,1} & d_{1,2} \\ d_{2,1} & d_{2,2} \\ \vdots & \vdots \\ d_{K,1} & d_{K,1} \end{bmatrix} \triangleq \begin{bmatrix} x^{\tau_{1,1}} & x^{\tau_{1,2}} \\ x^{\tau_{2,1}} & x^{\tau_{2,2}} \\ \vdots & \vdots \\ x^{\tau_{K,1}} & x^{\tau_{K,1}} \end{bmatrix} \tag{1}$$

and assume it to be known at all nodes before transmission.

For a message W to be transmitted at offset x^u and delayed by v time-slots, the resulting delayed message can be computed by $x^{u+v}W \mod (x^n - 1)$ for a period of n. For brevity, we denote $\mod (x^n - 1)$ as $\mod X^n$.

Encoding procedure: The codeword sent from $\mathcal{S}_k$ is encoded into the polynomial $v_k(x)$ by the encoding function e_k carrying the two messages $W_{k,1}$ and $W_{k,2}$:

$$e_k : (W_{k,1}, W_{k,2}) \rightarrow v_k(x) \tag{2}$$

where the input polynomial can be expressed as

$$v_k(x) = W_{k,1}x^{p_{k,1}} + W_{k,2}x^{p_{k,2}} \mod X^n \tag{3}$$

with parameter $p_{k,j}$ indicating the index of allocated time-slot for message $W_{k,j}$. The input polynomial vector is

$$\boldsymbol{v} = \begin{bmatrix} v_1(x) & v_2(x) & \cdots & v_K(x) \end{bmatrix} \tag{4}$$

Through the above PD-based model, each receiver obtains a polynomial denoted by $r_j(x), \forall j \in \{1, 2\}$. In the vector form,

$$\boldsymbol{r} = \begin{bmatrix} r_1(x) & r_2(x) \end{bmatrix} \equiv \boldsymbol{vD} \mod X^n \tag{5}$$

Decoding procedure: At $\mathcal{D}_j$, the received polynomial $r_j(x)$ is decoded to obtain the dedicated messages:

$$f_j : r_j(x) \rightarrow (\hat{W}_{1,j}, \hat{W}_{2,j}, \cdots, \hat{W}_{K,j}) \tag{6}$$

For this PD-based model, its DoF is defined by

$$DoF \triangleq \frac{M}{n} = \frac{2K}{n} \tag{7}$$

Remark: To focus on the core problem of PD-based IA, we assume unit-gain channels and zero noises as well as that in [9]. In fact, different channel gains among all source-destination pairs are also allowed in the system model. In this case, channel estimation and equalization techniques should be applied at each receiver before the above decoding procedure.

3. A General Perfect Cyclic IA Scheme

From the existing result of general XC [2], the maximum achievable DoF of $K\times 2$ XC is $2K/(K+1)$. In the PD-based model, we have $M = 2K$ different messages to convey and decode over a period of n time-slots. To achieve the theoretical DoF of $2K/(K+1)$, the minimal number of consecutive time-slots at the decoder side should be equal to $n = K + 1$. Otherwise, a larger period will decrease the achieved DoF, which makes the scheme suboptimal. In the following content, we only discuss this DoF-optimal configuration based on perfect cyclic IA over the above PD-based model.

Since each decoder needs K time-slots to separate its required K messages, only one time-slot is available for perfect cyclic IA. In detail, $\mathcal{D}_j$ should align its K undesired messages $W_{k,\bar{j}}, \forall k \in \{1, 2, \cdots, K\}, \bar{j} \neq j$ as interference into one of the total $K+1$ time-slots, which brings the **alignment conditions**:

$$\begin{aligned} & d_{k,j}x^{p_{k,\bar{j}}} \equiv d_{l,j}x^{p_{l,\bar{j}}} \mod X^{K+1} \\ \Leftrightarrow\ & x^{\tau_{k,j}+p_{k,\bar{j}}} \equiv x^{\tau_{l,j}+p_{l,\bar{j}}} \mod X^{K+1} \\ \Leftrightarrow\ & \tau_{k,j} + p_{k,\bar{j}} \equiv \tau_{l,j} + p_{l,\bar{j}} \mod K + 1 \end{aligned} \tag{8}$$

with $\forall k \neq l \in \{1, 2, \cdots, K\}$ and $j \neq \bar{j} \in \{1, 2\}$.

On the other hand, the desired K messages $W_{k,j}$ should be distinct with each other and occupy all the K time-slots except the one for cyclic IA, which introduces the **separability conditions** at $\mathcal{D}_j$:

$$\begin{aligned} & d_{k,j}x^{p_{k,j}} \not\equiv d_{l,j}x^{p_{l,j}} \mod X^{K+1} \\ \Leftrightarrow\ & x^{\tau_{k,j}+p_{k,j}} \not\equiv x^{\tau_{l,j}+p_{l,j}} \mod X^{K+1} \\ \Leftrightarrow\ & \tau_{k,j} + p_{k,j} \not\equiv \tau_{l,j} + p_{l,j} \mod K + 1 \end{aligned} \tag{9}$$

for the K messages with $\forall k \neq l \in \{1, 2, \cdots, K\}$, and

$$\begin{aligned} & d_{k,j}x^{p_{k,j}} \not\equiv d_{l,j}x^{p_{l,\bar{j}}} \mod X^{K+1} \\ \Leftrightarrow\ & x^{\tau_{k,j}+p_{k,j}} \not\equiv x^{\tau_{l,j}+p_{l,\bar{j}}} \mod X^{K+1} \\ \Leftrightarrow\ & \tau_{k,j} + p_{k,j} \not\equiv \tau_{l,j} + p_{l,\bar{j}} \mod K + 1 \end{aligned} \tag{10}$$

between the message and interference with $\forall k \neq l \in \{1, 2, \cdots, K\}$ and $j \neq \bar{j} \in \{1, 2\}$.

Theorem 1 summarizes our main contribution.

Theorem 1. *Perfect cyclic IA achieving the DoF of* $2K/(K+1)$ *for the* $K \times 2$ *PD-based X channels exists and a general scheme of [10] is given by the following* $K \times 2$ *PD matrix*

$$\boldsymbol{D}_{K_1} = \begin{bmatrix} x^{\tau_{1,1}} & x^{\tau_{1,2}} \\ x^{\tau_{2,1}} & x^{\tau_{2,2}} \\ \vdots & \vdots \\ x^{\tau_{K_1,1}} & x^{\tau_{K_1,2}} \\ x^{\tau_{K_1+1,1}} & x^{\tau_{K_1+1,2}} \\ x^{\tau_{K_1+2,1}} & x^{\tau_{K_1+2,2}} \\ \vdots & \vdots \\ x^{\tau_{K,1}} & x^{\tau_{K,2}} \end{bmatrix} = \begin{bmatrix} x^0 & x^{K_2} \\ x^1 & x^{K_2} \\ \vdots & \vdots \\ x^{K_1-1} & x^{K_2} \\ x^{K_1} & x^0 \\ x^{K_1} & x^1 \\ \vdots & \vdots \\ x^{K_1} & x^{K_2-1} \end{bmatrix} \tag{11}$$

where parameters $0 \leq K_1 \leq K$ *and* $K_2 \triangleq K - K_1$ *are two integers indicating the time-slot index for cyclic IA at* $\mathcal{D}_1$ *and* $\mathcal{D}_2$*, respectively, and the following transmitter polynomial*

$$v_k(x) = \begin{cases} W_{k,1} + x^{K_1-k+1}W_{k,2} \mod X^{K+1}, & \forall k \leq K_1 \\ W_{k,2} + x^{K-k+1}W_{k,1} \mod X^{K+1}, & \forall k > K_1 \end{cases} \tag{12}$$

Proof. The proof is divided into two parts. Firstly, we show the existence of perfect cyclic IA with a DoF of $2K/K + 1$. Then we show the general feature of the proposed scheme in contrast to that in [10].

From (5) with $n = K + 1$, at $\mathcal{D}_1$, we have

$$
\begin{aligned}
r_1(x) &\equiv \boldsymbol{v}\boldsymbol{D}_{K_1}(:,1) \mod X^{K+1} \\
&\equiv \sum_{k=1}^{K_1} v_k(x)x^{k-1} + \sum_{k=K_1+1}^{K} v_k(x)x^{K_1} \mod X^{K+1} \\
&\equiv \sum_{k=1}^{K_1} (W_{k,1} + x^{K_1-k+1}W_{k,2})x^{k-1} \\
&\quad + \sum_{k=K_1+1}^{K} (W_{k,2} + x^{K-k+1}W_{k,1})x^{K_1} \mod X^{K+1} \\
&\equiv \sum_{k=1}^{K_1} (x^{k-1}W_{k,1} + x^{K_1}W_{k,2}) \\
&\quad + \sum_{k=K_1+1}^{K} (x^{K_1}W_{k,2} + x^{K+K_1-k+1}W_{k,1}) \mod X^{K+1} \\
&\equiv \sum_{k=1}^{K_1} x^{k-1}W_{k,1} + \sum_{k=1}^{K} x^{K_1}W_{k,2} \\
&\quad + \sum_{k=K_1+1}^{K} x^{K+K_1-k+1}W_{k,1} \mod X^{K+1} \qquad (13)
\end{aligned}
$$

The second term in the last step shows that the alignment conditions (8) are satisfied, since all the interference messages $W_{k,2}, \forall k \in \{1, 2, \cdots, K\}$ are aligned into the time-slot K_1, i.e., $d_{k,1}x^{p_{k,2}} \equiv d_{l,1}x^{p_{l,2}} \equiv x^{K_1} \mod X^{K+1}$, with $\forall k \neq l \in \{1, 2, \cdots, K\}$. The first term indicates that the desired message $W_{k,1}$ is exclusively located in time-slot $k-1$, $\forall k \in \{1, 2, \cdots, K_1\}$, while the third term indicates that the desired message $W_{k,1}$ is exclusively located in time-slot $K+K_1-k+1$, $\forall k \in \{K_1+1, K_1+2, \cdots, K\}$. So the separability conditions (9)–(10) are also satisfied.

Similarly, at $\mathcal{D}_2$ the received polynomial is

$$
\begin{aligned}
r_2(x) &\equiv \boldsymbol{v}\boldsymbol{D}_{K_1}(:,2) \mod X^{K+1} \\
&\equiv \sum_{k=1}^{K_1} v_k(x)x^{K_2} + \sum_{k=K_1+1}^{K} v_k(x)x^{k-K_1-1} \mod X^{K+1} \\
&\equiv \sum_{k=1}^{K_1} (W_{k,1} + x^{K_1-k+1}W_{k,2})x^{K_2} \\
&\quad + \sum_{k=K_1+1}^{K} (W_{k,2} + x^{K-k+1}W_{k,1})x^{k-K_1-1} \mod X^{K+1} \\
&\equiv \sum_{k=1}^{K_1} (x^{K_2}W_{k,1} + x^{K-k+1}W_{k,2}) \\
&\quad + \sum_{k=K_1+1}^{K} (x^{k-K_1-1}W_{k,2} + x^{K-K_1}W_{k,1}) \mod X^{K+1} \\
&\equiv \sum_{k=1}^{K_1} x^{K-k+1}W_{k,2} + \sum_{k=1}^{K} x^{K_2}W_{k,1} \\
&\quad + \sum_{k=K_1+1}^{K} x^{k-K_1-1}W_{k,2} \mod X^{K+1} \qquad (14)
\end{aligned}
$$

The second term in the last step indicates that the alignment conditions (8) are satisfied by IA at the time-slot K_2. The first term indicates that the desired message $W_{k,2}$ is exclusively located in time-slot $K-k+1$, $\forall k \in \{1, 2, \cdots, K_1\}$, while the third term indicates that the desired message $W_{k,2}$ is exclusively located in time-slot $k-K_1-1$, $\forall k \in \{K_1+1, K_1+2, \cdots, K\}$. Thus, the separability conditions (9)–(10) are still satisfied.

In summary, $2K$ different messages are collected over $K+1$ time-slots, which indicates the DoF of $2K/(K+1)$ is achieved by the proposed scheme. So perfect cyclic IA exists for the PD-based $K\times 2$ X channels. Moreover, the above deduction also shows that K_1 and K_2 indicate the time-slot index of cyclic IA at $\mathcal{D}_1$ and $\mathcal{D}_2$, respectively.

Then what left is the generality proof. For any given K, [10] gives one feasible PD pattern. In contrast, by adjusting K_1 from 0 to K, the proposed scheme can provide $K+1$ feasible PD patterns. Here we show that the one in [10] is just a special case of this work. This can be verified by setting

$$
\begin{cases} K_1 = N,\ K_2 = N & \text{even K: } \forall K = 2N \\ K_1 = N,\ K_2 = N-1 & \text{odd K: } \forall K = 2N-1 \end{cases} \qquad (15)
$$

where $N \triangleq \lfloor (K+1)/2 \rfloor$ is the floored integer on $(K+1)/2$. For example, if $K=6$, we have $N = \lfloor 7/2 \rfloor = 3$ and $K = 2N$, while if $K=5$, we have $N = \lfloor 6/2 \rfloor = 3$ and $K = 2N-1$. Next we take a closer look. With even $K = 2N$, the above setting $K_1 = K_2 = N$ gives the following PD matrix

$$
\boldsymbol{D}_N = \begin{bmatrix} x^0 & x^N \\ x^1 & x^N \\ \vdots & \vdots \\ x^{N-1} & x^N \\ x^N & x^0 \\ x^N & x^1 \\ \vdots & \vdots \\ x^N & x^{N-1} \end{bmatrix}
$$

which results in the equivalent PD matrix expression given by Eq. (18) in [10]. Correspondingly, the transmitter polynomial (12) becomes

$$
v_k(x) = \begin{cases} W_{k,1} + x^{N-k+1}W_{k,2} \mod X^{K+1}, & \forall k \leq N \\ W_{k,2} + x^{K-k+1}W_{k,1} \mod X^{K+1}, & \forall k > N \end{cases}
$$

which shows the same scheduling message with Eq. (19) in [10]. With odd $K = 2N-1$, the above setting $K_1 = N$, $K_2 = N-1$ gives the following PD matrix

$$
\boldsymbol{D}_N = \begin{bmatrix} x^0 & x^{N-1} \\ x^1 & x^{N-1} \\ \vdots & \vdots \\ x^{N-1} & x^{N-1} \\ x^N & x^0 \\ x^N & x^1 \\ \vdots & \vdots \\ x^N & x^{N-2} \end{bmatrix}
$$

which results in the equivalent PD matrix expression given by Eq. (21) in [10]. The corresponding transmitter polynomial (12) is

$$v_k(x) = \begin{cases} W_{k,1} + x^{N-k+1} W_{k,2} \mod X^{K+1}, & \forall k \le N \\ W_{k,2} + x^{K-k+1} W_{k,1} \mod X^{K+1}, & \forall k > N \end{cases}$$

which shows the same scheduling message with Eq. (22) in [10]. Therefore, we have proven that the scheme in [10] just a special case of this paper. In other words, the proposed scheme is a generalization of that in [10]. This completes the proof. □

More discussions are provided below for better understanding of the features of this general scheme.

The PD matrix (11) can be divided into two parts. The upper part includes the first K_1 rows, which has the same PD x^{K_2} to $\mathcal{D}_2$ and increasing PD x^0 to x^{K_1-1} from $\mathcal{S}_1$ to $\mathcal{S}_{K_1}$. The lower part includes the last K_2 rows, which has the same PD x^{K_1} to $\mathcal{D}_1$ and increasing PD x^0 to x^{K_2-1} from $\mathcal{S}_{K_1+1}$ to $\mathcal{S}_K$. The proportion of the upper part is determined by K_1. By adjusting K_1 from 0 to K, the PD matrix has total $K+1$ patterns. Specially, if $K_1 = 0$ or equivalently $K_2 = K$, the upper part will be an empty matrix without dimension since $K_1 - 1 = -1$. In this case, the PD matrix $\boldsymbol{D}_0$ will only have the lower part as shown on the left of (16). On the other hand, if $K_1 = K$ or equivalently $K_2 = 0$, the lower part will be an empty matrix without dimension since $K_2 - 1 = -1$. In this case, the PD matrix $\boldsymbol{D}_K$ will only have the upper part as shown on the right of (16). Thus we have

$$\boldsymbol{D}_0 = \begin{bmatrix} x^0 & x^0 \\ x^0 & x^1 \\ \vdots & \vdots \\ x^0 & x^{K-1} \end{bmatrix}, \quad \boldsymbol{D}_K = \begin{bmatrix} x^0 & x^0 \\ x^1 & x^0 \\ \vdots & \vdots \\ x^{K-1} & x^0 \end{bmatrix} \tag{16}$$

for $K_1 = 0$ and $K_2 = 0$, respectively. Accordingly, the input polynomial is

$$v_k(x) = \begin{cases} W_{k,2} + x^{K-k+1} W_{k,1}, & K_1 = 0 \\ W_{k,1} + x^{K-k+1} W_{k,2}, & K_2 = 0 \end{cases} \tag{17}$$

And the received polynomials are

$$\begin{bmatrix} r_1(x) \\ r_2(x) \end{bmatrix} \equiv \begin{bmatrix} \sum_{k=1}^{K} x^0 W_{k,2} + \sum_{k=1}^{K} x^{K-k+1} W_{k,1} \\ \sum_{k=1}^{K} x^K W_{k,1} + \sum_{k=1}^{K} x^{k-1} W_{k,2} \end{bmatrix} \mod X^{K+1}$$

for $K_1 = 0$, and

$$\begin{bmatrix} r_1(x) \\ r_2(x) \end{bmatrix} \equiv \begin{bmatrix} \sum_{k=1}^{K} x^{k-1} W_{k,1} + \sum_{k=1}^{K} x^K W_{k,2} \\ \sum_{k=1}^{K} x^{K-k+1} W_{k,2} + \sum_{k=1}^{K} x^0 W_{k,1} \end{bmatrix} \mod X^{K+1}$$

for $K_2 = 0$, respectively. We can see that the above PD matrices, input and received polynomials with $K_2 = 0$ are equal to that with $K_1 = 0$ if we exchange the role of $\mathcal{D}_1$ and $\mathcal{D}_2$. So the above cyclic IA scheme exhibits a kind of symmetric property between K_1 and K_2, i.e., a symmetry within the range of K_1 from 0 to K.

On the encoder side, we have some observations from (12). When $k \le K_1$, message $W_{k,1}$ always occupies the first time slot with index 0, while the other one $W_{k,2}$ is allocated in the time slot $K_1 - k + 1$. For example, $W_{1,2}$ uses time slot K_1, and $W_{K_1,2}$ uses time slot 1. When $k > K_1$, message $W_{k,2}$ always occupies the time slot 0, while the other one $W_{k,1}$ is allocated in the time slot $K - k + 1$. For example, $W_{K_1+1,1}$ uses time slot $K - K_1$, and $W_{K,1}$ uses time slot 1. **Scheduling window** is defined as the spanned time-slots from the first message to the last one. When $k \le K_1$, the scheduling window at $\mathcal{S}_k$ is from time slot 0 to time slot $K_1 - k + 1$, among which $\mathcal{S}_1$ has the longest scheduling window of $1 + K_1 - 1 + 1 = K_1 + 1$ time slots. When $k > K_1$, the scheduling window at $\mathcal{S}_k$ is from time slot 0 to time slot $K - k + 1$, among which $\mathcal{S}_{K_1+1}$ has the longest scheduling window of $1 + K - (K_1 + 1) + 1 = 1 + K - K_1 = K_2 + 1$ time slots. So the maximal length of scheduling window denoted by L is $\max(K_1 + 1, K_2 + 1) = \max(K_1, K_2) + 1$. The above deduction can be mathematically expressed as below

$$\begin{aligned} L &= \max\left(1 + (K_1 - k + 1)|_{k=1}^{K_1}, 1 + (K - k + 1)|_{k=K_1+1}^{K}\right) \\ &= \max(K_1, K_2) + 1 \end{aligned} \tag{18}$$

Tables 1 and 2 show the detailed evaluation of K_1, K_2 and L for even and odd K, respectively. From Table 1 we can see that there are total $M = N + 1$ kinds of different L for even $K = 2N$, among which the least length occurs at $K_1 = N$. From Table 2 we know that there are total $M = N$ kinds of different L for odd $K = 2N - 1$, among which the least length occurs at $K_1 = N - 1$ and $K_1 = N$. So there are $M = \lfloor K/2 \rfloor + 1$ different values of L for a general K. The minimum of L is $N + 1 = \lceil K/2 \rceil + 1$ for both tables, which shows the least length of scheduling window for the proposed scheme. All other values of L from $N + 2$ to $K + 1$ have two combinations of K_1 and K_2. The maximum of L is $K + 1$, which occurs at $K_1 = 0$ or $K_1 = K$. From the implement viewpoint, a shorter scheduling window implies less complexity, since the shift register can use a smaller length. By choosing proper value of K_1, the proposed scheme can be adaptive to variant implementation requirements.

4. Feasibility in the Euclidean Space

We analyze the feasibility of the above PD-based scheme by giving the conditions of node placement in the Euclidean space. For simplicity, constant propagation speed (v) is assumed. Denote the distance between $\mathcal{D}_1$ and $\mathcal{D}_2$ as a. The distance between $\mathcal{S}_1$ and $\mathcal{D}_1$ is initialized as $d_0 = v\tau_0$, where τ_0 is any positive value indicating the reference PD. The distance increase step is denoted by $\Delta d = vs\Delta\tau > 0$, where s is a scaling factor and $\Delta\tau$ is the PD gap which has been normalized in the above model.

Figure 1 is used for demonstration. Denote the circle/sphere centered at $\mathcal{D}_j$ with radius d by $O_j(d)$. The most inner circles/spheres have a radius of d_0, while the increase step of radius is Δd. The most outer circle/sphere around $\mathcal{D}_1$ and $\mathcal{D}_2$ has a radius of $d_0 + K_1\Delta d$ and $d_0 + K_2\Delta d$, respectively. The positions of all source nodes are also partially

Table 1 The length of scheduling windows with even $K = 2N$.

K_1	0	1	$\cdots$	$N-1$	N	$N+1$	$\cdots$	$K-1$	K
K_2	K	$K-1$	$\cdots$	$N+1$	N	$N-1$	$\cdots$	1	0
L	$K+1$	K	$\cdots$	$N+2$	$N+1$	$N+2$	$\cdots$	K	$K+1$

Table 2 The length of scheduling windows with odd $K = 2N - 1$.

K_1	0	1	$\cdots$	$N-2$	$N-1$	N	$N+1$	$\cdots$	$K-1$	K
K_2	K	$K-1$	$\cdots$	$N+1$	N	$N-1$	$N-2$	$\cdots$	1	0
L	$K+1$	K	$\cdots$	$N+2$	$N+1$	$N+1$	$N+2$	$\cdots$	K	$K+1$

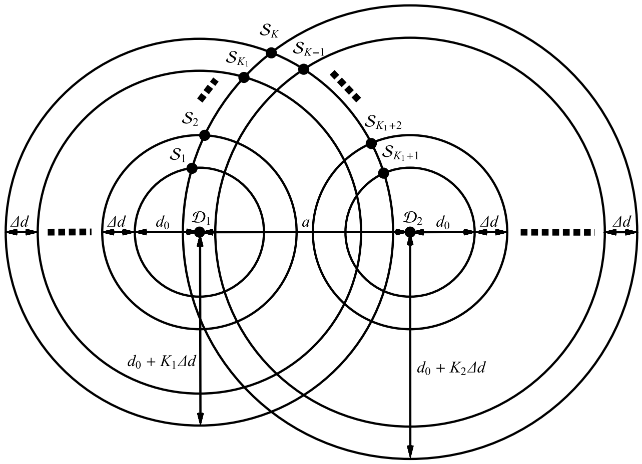

Fig. 1 Feasibility demonstration for $K \times 2$ XC with arbitrary integers $0 \le K_1 \le K$ and $K_2 = K - K_1$.

shown by the cross points of related circles/spheres. Figure 1 only shows one possible candidate choice of the source nodes in the upper part. Indeed, the intersection points of the relative circles in the lower part are also feasible for the source nodes in the two-dimensional Euclidean space. So there might be many combinations of the node placement. In the three-dimensional Euclidean space, the intersection of the relative spheres is a circle. Because any point on the intersection circle is feasible for a source node, the number of combinations of the node placement can be large enough as long as the intersection circle has a positive radius and enough high resolution. When $K_1 = 0$ or $K_2 = 0$, there will be only one circle/sphere around $\mathcal{D}_1$ or $\mathcal{D}_2$ respectively, on which all source nodes are located.

The feasibility condition of the proposed scheme requires that the most outer circles/spheres centered at $\mathcal{D}_1$ must have at least one intersection point with the most inner circles/spheres centered at $\mathcal{D}_2$, and vice versa. Mathematically, we have: $\forall K_1 = 0, \cdots, K$ and $\forall K_2 = K - K_1$

$$\begin{aligned} O_1(d_0 + K_1\Delta d) \cap O_2(d_0) &\neq \emptyset \\ O_1(d_0) \cap O_2(d_0 + K_2\Delta d) &\neq \emptyset \end{aligned} \tag{19}$$

which can be further expressed with a as

$$\begin{aligned} a - d_0 \;\le\; d_0 + K_1\Delta d \;\le\; a + d_0 \\ a - d_0 \;\le\; d_0 + K_2\Delta d \;\le\; a + d_0 \end{aligned} \tag{20}$$

After some computations, the above feasibility condition (20) can be simplified as

$$\max(K_1, K_2)\Delta d \le a \le 2d_0 + \min(K_1, K_2)\Delta d \tag{21}$$

Due to the relationship $K_2 = K - K_1$, we can see that a smaller $\max(K_1, K_2)$ has a wider range of a. Especially, when $K_1 = 0$ or $K_2 = 0$, with fixed Δd the narrowest range of a is

$$K\Delta d \le a \le 2d_0 \tag{22}$$

which also implies that $\Delta d \le 2d_0/K$. On the other hand, with fixed Δd the widest range of a is

$$N\Delta d \le a \le 2d_0 + N\Delta d \tag{23}$$

for even $K = 2N$ with $K_1 = K_2 = N$ and

$$N\Delta d \le a \le 2d_0 + (N - 1)\Delta d \tag{24}$$

for odd $K = 2N - 1$ with $K_1 = N$ or $K_2 = N$. Since there is no other constraint on Δd in (23), theoretically it can be any positive value. With proper d_0 and Δd, the distance between $\mathcal{D}_1$ and $\mathcal{D}_2$ can be determined as required. However, with odd $K = 2N - 1$, the feasibility condition (24) implies that $\Delta d \le 2d_0$, which limits the upper bound on a to be $2Nd_0 = (K + 1)d_0$. In this sense, even K is more preferred than odd K. Other choice of K_1 between 0 and K provides a trade-off between the narrowest and widest ranges of a. By flexibly adjusting K_1, d_0 and Δd (equivalently the frame length $\Delta\tau$), the feasibility condition (21) can be widely practical.

Examples: For underwater acoustic applications, we can set parameters as $v = 1500$m/s and $d_0 = 1000$m. Two examples are given below.

(i) For a 10×2 XC with $K_1 = 0$ or $K_1 = 10$, the feasibility condition is $10\Delta d \le a \le 2d_0 = 2000$m, which gives the narrowest range of a and implies that $\Delta d \le 200$m. On the other hand, with $K_1 = 5$, the feasibility condition is $5\Delta d \le a \le 2d_0 + 5\Delta d = 2000 + 5\Delta d$, which gives the widest range of a with arbitrarily value of $\Delta d > 0$.

(ii) For a 9×2 XC with $K_1 = 0$ or $K_1 = 9$, the feasibility condition is $9\Delta d \le a \le 2d_0 = 2000$m, which gives the narrowest range of a and implies that $\Delta d \le 2000/9$m. On the other hand, with $K_1 = 5$ or $K_2 = 5$, the feasibility condition is $5\Delta d \le a \le 2d_0 + 4\Delta d = 2000 + 4\Delta d$, which gives the widest range of a with $\Delta d \le 2d_0 = 2000$m.

Moreover, when a suboptimal n (i.e. $n > K + 1$) is considered, we can expect that the feasibility can be greatly improved, since there will be more feasible patterns supported and the alignment conditions can be relaxed to some extent. For example, with $n = K + 2$, now we have two time-slots for cyclic IA of the K undesired messages, which has many combinations of possible interference grouping for cyclic IA at each destination node, in contrast with the only one combination with $n = K+1$. This provides a trade-off method between the multiplexing gain and feasibility in the Euclidean space. When no perfect cyclic IA exists such as in the 3-user X-networks shown in [12], this approach can be expected to make a feasible suboptimal scheme. In fact, when more time slots are involved, more messages besides the original ones are also possible, which compensates the DoF loss in some extent. For example, with one extra time slot, by allowing one more message the maximum DoF of 10/6=5/3 instead of 9/6=3/2 is achieved based on cyclic IA for the three-user X channel in [13].

5. Conclusion

We extended the approach of perfect cyclic IA by PD to $K \times 2$ X channels and proposed a general scheme, which is optimal to achieve the DoF of $2K/(K + 1)$. The position of IA can be arbitrarily determined by the parameter K_1 from time-slot 0 to K along with the $K + 1$ patterns. Feasibility analysis shows that it is widely practical in the Euclidean space. This work generalizes the scope of cyclic IA for X channels in temporal domain.

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