LETTER

Phase Center Calibration for UWB Phase Interferometer Direction Finding by Virtual Baseline

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SUMMARY Phase interferometer using baseline composed by uniform linear array (ULA) with stable phase center for estimating the angle of arrival (AOA) is always employed in the direction finding (DF) system. However, the phase center of antenna element could vary with the incident angle, frequency, multipath and so on. To deal with these problems, a novel method is proposed in this paper to calibrate the phase center over ultra-wideband (UWB). Meanwhile, the restrictions of this method are discussed. Numerical simulations reveal that higher accuracy and larger unambiguous angle range can be obtained by the proposed method.

key words: *phase interferometer, UWB, virtual baseline, AOA, the phase center, calibration*

1. Introduction

Phase interferometer direction finding (DF) system with high accuracy is widely used in radar, sonar and wireless communication, where phase ambiguity and accuracy are important problems that have to be addressed to determine the angle of arrival (AOA). So the researches of phase interferometer currently focus on algorithms of ambiguity resolution and accuracy analysis [1]–[3]. However, accurate baseline for calculating phase difference is a necessary condition for these researches. The baseline is the line connecting the phase centers of DF antennas. When the antenna phase center changes, the baseline also changes, which seriously reduces the accuracy of phase interferometer.

Schupler and others [4] first proposed to calibrate antenna phase center in the microwave anechoic chamber. This method is not only time-consuming and laborious, but also difficultly for implementation. Some literatures [5], [6] have proposed detailed methods about phase center measurements, which also requiring adjust the location of measuring antenna constantly, making it very complicated and inconvenient especially in higher frequency band.

As is revealed in [8], the idea of virtual antenna with controllable location by interpolation among uniform linear antenna array (ULA) is proposed to compensate for the doppler spread interference. This method is also employed in [9] to guarantee the optimal capacity of the MIMO communication systems. Inspired by the idea of the virtual antenna using interpolation with antenna array, virtual baseline with flexible length based on ULA is proposed for direction finding (DF) system using ultra-wideband (UWB) interferometer, which can almost calibrate the fluctuations of the antenna phase center and provide unambiguous AOA during detecting.

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2. Virtual Baseline Algorithm

2.1 Baseline Error

Phase center is defined in the IEEE standards as: "The location of a point associated with an antenna such that, if it is taken as the center of a sphere whose radius extends into the far-field, the phase of a given field component over the surface of the radiation sphere is 'essentially' constant, at least over the portion of the surface where the radiation is significant." In practical engineering, phase center offset (PCO) and phase center variation (PCV) lead to the antenna reference point (ARP), which is called as rotation center or geometrical center of antenna, deviating from the antenna phase center [6]. Analytical results show that, different frequencies, different incident angles, different material and multipath, all could make great influence on antenna phase center [7].

According to the above analysis, for an ULA, all the phase centers of antenna elements would deviate from their ARPs in practice. In the application of DF system, high accuracy of angle can be obtained with long baseline, thus the baseline error of ULA is determined by the antenna elements at both ends of the baseline. Now, as shown in Fig. 1, for a phase interferometer DF system equipped with ULA, and the antenna elements at both ends are marked as A and B. Suppose that the phase centers of A and B deviate from their ARPs. Assume L is the baseline determined by their ARP, and L' is the baseline of their phase centers. The baseline error is $\Delta L' = |L - L'|$. It is known that, the AOA error $\Delta\theta$ in this ULA can be derived by the phase difference error $\Delta\varphi$ caused by inner noise, the frequency error $\Delta\lambda$ and the baseline error $\Delta L'$ as referred to in Eq. (1), where θ is the incident angle, $\Delta\varphi$ is the phase difference, λ is the wavelength.

$$\Delta\theta = \frac{\lambda}{2\pi L\cos\theta}\Delta\varphi + \frac{\tan\theta}{\lambda}\Delta\lambda - \frac{\tan\theta}{L}\Delta L' \tag{1}$$

Therefore, the parameter $\Delta L'$ referred to in Eq. (1) deteriorates the accuracy of phase interferometer for an ULA.

2.2 The Algorithm of Phase Center Calibration

To deal with the problem mentioned above, an interpo-

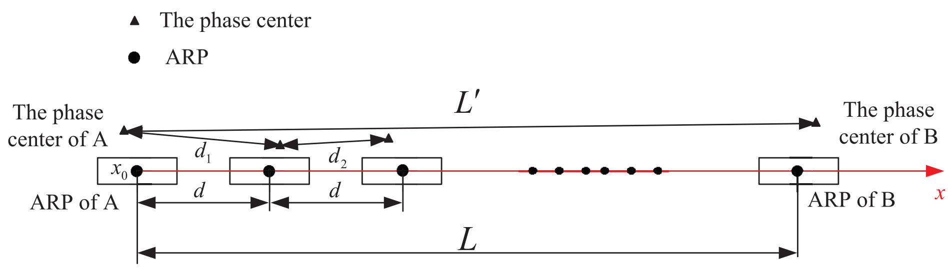


Fig. 1 Baseline error of ULA.

lation algorithm is employed to construct virtual antennas, and two virtual antennas make up one virtual baseline. Assume that the number of the physical antennas is N, the number of the virtual antennas is two, and the phase center coordinate of the first antenna A rray is x_0. Then, the phase center coordinates of other antennas are $[x_0 + d_1, x_0 + d_1 + d_2, \ldots, x_0 + d_1 + d_2 \ldots + d_{N-1}]$, where d_i, $i = 1, \ldots, N-1$, is the distance from the phase center of the i-th antenna to the previous antenna and the coordinate of the virtual antennas can be denoted by $[x_0 + d_{V1}, x_0 + d_{V2}]^T$, where T denotes the transpose and d_{V1}, d_{V2} are the distance from the phase centers of the two virtual antennas $V1$ and $V2$ to x_0. Let P denote the cross-correlation vector between the received signal vector $r(t) = [r_0(t), r_1(t), \cdots, r_{N-1}(t)]^{\mathrm{T}}$, and the desired one $r'(t) = [r'_0(t), r'_1(t)]$.

$$\begin{aligned} \mathrm{P} &= \mathrm{E}\left[\mathrm{r}(t)\,\mathrm{r}'(t)^*\right] = \left[\mathrm{P}_{ij}\right] \\ &= \left[\mathrm{J}_0\left(\frac{2\pi i d - d_k}{\lambda}\right)\right], 1 \le i \le \mathrm{N}, 1 \le j \le \mathrm{M} \end{aligned} \tag{2}$$

where J_0 denotes the first-class Bessel function with zero order. It should be noted that d_i which are the phase center spacing in ULA are not constant. They vary with the change of phase center of the antenna element. Let Δd_i denoting the spacing error between two ARPs whose spacing are d and d_i. So $d_i = d + \Delta d_i$.

The signal of the virtual antennas $\mathrm{r}'(t)$ with minimum mean square error (MMSE) interpolation can be obtained by [9], [10],

$$\sigma^2{}_{\min} = J_0(0) - P^T R^{-1} P \tag{3}$$

Considering a DF system with $d = 0.45\lambda$ [5], the parameter of root mean square error (RMSE) denoted by $\sigma_{\min}$, representing the error between the signal of virtual antenna estimated by improved interpolation algorithms and the signal received by real antenna in the same location, is used here to evaluate the performance of this algorithm [9], [10]. Assume that the amplitudes of received signals are normalized, the antenna elements in ULA have the same received signal-to-noise ratio (SNR), the directions of incident waves are uniformly distributed, the frequency is 18 GHz, the number of virtual antennas is 2, $d = 7.5\,mm$, Δd_i is Gaussian distribution, the probability of $\Delta d_i \in \left[-3\sigma_{d_i}, 3\sigma_{d_i}\right]$ is 99.74%, $3\sigma_{d_i} = 10\%d$, $d_{V1} = 15\lambda$ and $d_{V2} = 15.5\lambda$. The results is obtained by 1000 times Monte Carlo simulations. As showed in Fig. 2(a) where the RMSE with different number of physical antennas is depicted, it can be found that the RMSE of two interpolated signals obtained by virtual antennas are all below 10^{-2} when $N \ge 36$, and as shown in Fig. 2(b), the RMSE can be controlled less than 10^{-2} when d_{V1} is varying from $15d$ to $26d$.

Therefore, two virtual antennas with flexible length can be obtained by improved interpolation algorithm. Moreover, the virtual baseline can be obtained by $L_V = d_{V2} - d_{V1} = 0.5\lambda$. Compared with the interpolation algorithm in reference [8], [9], the improved interpolation algorithm proposed in this paper takes into account the influence of change of antenna phase center.

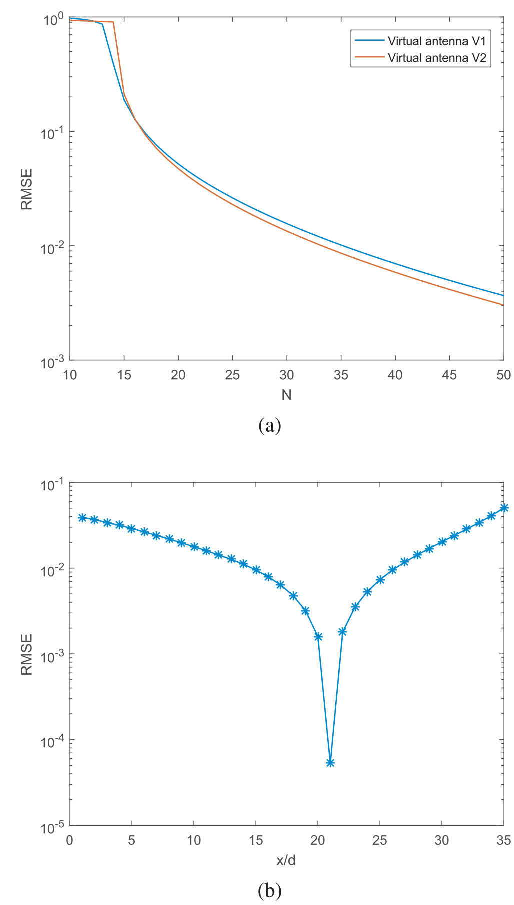


Fig. 2 Evaluation of RMSE. (a) RMSE with different N. (b) RMSE with $d = 7.5\,mm$ and $N = 36$.

In this part, virtual baseline L_V based on above description is used to calibrate $\Delta L'$ to improve accuracy of AOA. Firstly, accurate incident angle θ of target can be evaluated by Eq. (4) because virtual baseline $L_V = 0.5\lambda$ can provide unambiguous AOA. Then φ can be calculated by known $L = (N-1)\,d$ and θ, and φ' is measured by the baseline L' with A and B. So $\Delta L'$ can be achieved by Eq. (5),

$$\varphi_V = \frac{2\pi L_V}{\lambda}\sin\theta = \pi\sin\theta \qquad (4)$$

$$|\varphi - \varphi'| = \frac{2\pi\,|L - L'|}{\lambda}\sin\theta = \frac{2\pi\Delta L'}{\lambda}\sin\theta \qquad (5)$$

2.3 Restrictions

The proposed phase center calibration method is restricted to the received SNR. First, the requirement for obtaining an unambiguous AOA is that the AOA error $\Delta\theta_V$ of the virtual baseline should be within a half of the maximal unambiguous angle range θ_{Vu} as expressed in Eq. (6). And AOA error of virtual baseline is also obtained by phase difference error $\Delta\varphi_V$, the frequency error $\Delta\lambda$ and the virtual baseline error ΔL_v of virtual baseline L_v, and can be expressed in Eq. (7).

$$\Delta\theta_V < \frac{\theta_{Vu}}{2} \qquad (6)$$

$$\Delta\theta_V = \frac{\lambda}{2\pi L_V\cos\theta}\Delta\varphi_V + \frac{\tan\theta}{\lambda}\Delta\lambda - \frac{\tan\theta}{L_V}\Delta L_V \qquad (7)$$

While it is notable that $\Delta\lambda$ can be ignored due to the fact that $\Delta\lambda/\lambda$ in Eq. (7) is too small. Assume that $\Delta\phi$ and ΔL_V are all Gaussian distribution. So the probability of $\Delta\varphi_V \in \left[-3\sigma_{\varphi_V}, 3\sigma_{\varphi_V}\right]$ and $\Delta L_V \in [-3\sigma_{L_V}, 3\sigma_{L_V}]$ are all 99.74%. Then, let $\Delta\varphi_V = 3\sigma_{\varphi_V}$ and $\Delta L_V = 3\sigma_{L_V}$, the following equation can be derived,

$$\frac{\lambda}{2\pi L_v\cos\theta}3\sigma_{\varphi_V} - \frac{tan\theta}{L_v}3\sigma_{L_V} < arcsin\left(\frac{\lambda}{2(L_v + 3\sigma_{L_V})}\right) \qquad (8)$$

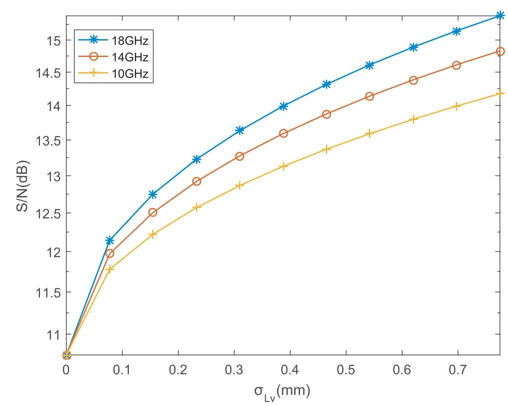
where, $\sigma_{\varphi_V} = \sqrt{N/(2SNR)}$, which is $\sqrt{N}$ times of the phase difference error ($\sqrt{1/(2SNR)}$) obtained by single antenna element [11]. So the minimum SNR can be calculated in Eq. (9) as,

$$SNR > \left(\frac{3\lambda\sqrt{N}}{2\sqrt{2}\pi cos\theta L_v\left(arcsin\frac{\lambda}{2\left(L_v+3\sigma_{L_V}\right)} + \frac{3\sigma_{L_V}tan\theta}{L_v}\right)}\right)^2 \qquad (9)$$

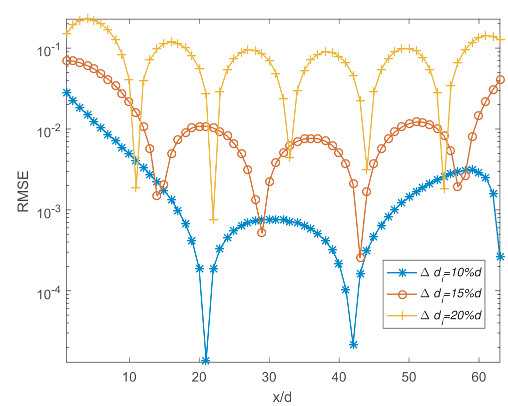
Moreover, the virtual baseline error ΔL_v which is different from $\Delta L'$ is determined by the RMSE of the signals obtained by the two virtual antennas using the interpolation algorithm. And, with normalized amplitude, the RMSE of signals obtained by the two virtual antennas can be derived by phase difference. Thus, σ_{L_V} can be obtained by Eq. (10) as,

Table 1 Simulation parameters.

Parameter	Value
Frequency	10 to 18 GHz
SNR	20 dB
Maximal wavelength λ_{max}	30 mm
ARP's spacing d	7.5 mm
Number of element	64
Coordinate range of virtual antenna	52.5 mm to 472.5 mm
Incident angle θ	5°
Angle measuring range θ_u	±30°
Maximal σ_{L_V}	0.775 mm
Maximal $\Delta L'$	19.4 mm



(a)



(b)

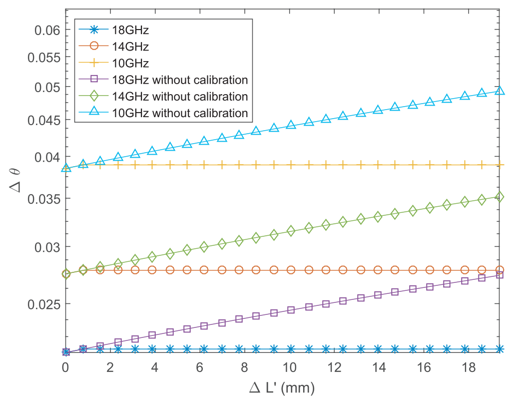
Fig. 3 Restrictions for virtual baseline. (a) required SNR with different frequencies f = 10 to 18 GHz. (b) RMSE with different Δd_i.

$$\frac{2\pi\sigma_{L_V}}{\lambda}\sin\theta \le \sqrt{2}\cdot 10^{-2} \qquad (10)$$

3. Performance Evaluation

3.1 Simulation Parameters

Due to the small spacing between antennas, the mutual coupling among the elements cannot be neglected. Fortunately, the mutual coupling elimination method has been discussed in [8]. It will not be discussed here. The simulation parameters are listed in Table 1.



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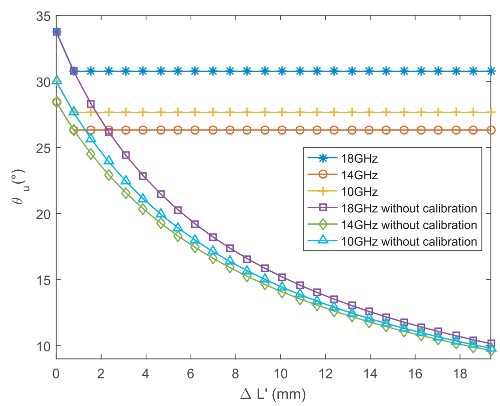

(b)

Fig. 4 Capacity with and without calibration. (a) accuracy of AOA with different frequencies $f = 10$ to 18 GHz. (b) angle measuring range with different frequencies $f = 10$ to 18 GHz.

3.2 Restrictions

As shown in Fig. 3(a), the required minimum SNR for obtaining unambiguous AOA increases with the RMSE of the virtual baseline error σ_{L_V} and the working frequency. Also, it can be found from Fig. 3(b) that, for 18 GHz frequency, spacing error $3\sigma_{d_i} = 10\%d$, $15\%d$ and $20\%d$ affects the RMSE of the signals obtained by virtual antennas. The position range of virtual antenna decreases with the increasing spacing error when RMSE is required to be less than 10^{-2}.

3.3 Performance

Figure 4 compares the DF performance of ULA with and without the proposed method. In Fig. 4(a), without calibration, $\Delta L'$ of antenna elements at both ends degrade the accuracy of AOA. The larger the $\Delta L'$ is, the worse the accuracy will be. With phase center calibration, the deterioration caused by $\Delta L'$ will be mitigated.

In Fig. 4(b), actual baselines are selected to obtain angle measuring range θ_u of different operating frequencies. For example, if the operating frequency is 14 GHz, the actual baseline is $3d$, which is closest to the wavelength of 14 GHz. Without calibration, the angle measuring range θ_u is reduced with the baseline error $\Delta L'$ of actual baseline. So it is not possible to use actual baselines to obtain widely unambiguous angular values. With the proposed method, the virtual baseline can provide wider range.

4. Conclusion

Stable phase center is the foundation of DF system with phase interfering method. Inspired by the idea of antenna interpolation, an improved method of virtual baseline is proposed to obtain unambiguous AOA and calibrate the phase center of long baseline in real-time for UWB DF system. The required SNR of this method is calculated. Simulation results reveal that higher angle accuracy and larger unambiguous angle range for AOA measurement can be obtained compared with that have no calibration method.

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