LETTER

# QoS-Constrained Robust Beamforming Design for MIMO Interference Channels with Bounded CSI Errors

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**SUMMARY** Constrained by quality-of-service (QoS), a robust transceiver design is proposed for multiple-input multiple-output (MIMO) interference channels with imperfect channel state information (CSI) under bounded error model. The QoS measurement is represented as the signal-to-interference-plus-noise ratio (SINR) for each user with single data stream. The problem is formulated as sum power minimization to reduce the total power consumption for energy efficiency. In a centralized manner, alternating optimization is performed at each node. For fixed transmitters, closed-form expression for the receive beamforming vectors is deduced. And for fixed receivers, the sum-power minimization problem is recast as a semi-definite program form with linear matrix inequalities constraints. Simulation results demonstrate the convergence and robustness of the proposed algorithm, which is important for practical applications in future wireless networks.

***key words:*** *MIMO interference channel, QoS constraints, bounded CSI error, robust transceiver design*

## 1. Introduction

The multiple-input multiple-output (MIMO) technology significantly increases the spectrum efficiency of wireless communication. Unlike the conventional multiuser channels where there is only transmitter for the downlink or only one receiver for the uplink, there are multiple transmitters and receivers in the interference channels. This brings new challenges for the transceiver design, which is adaptive to the multi-cell scenario. With multiple antenna support, the request for better performance has led researchers to jointly optimize the transceiver design in the MIMO scenario [1], [2].

Perfect channel state information (CSI) were often assumed at beginning. Joint design of linear transceivers were considered for the multi-user MIMO interference channels with quality-of-service (QoS) constraints in [1] and for the MIMO interfering broadcast channel in [2] to achieve max-min fairness, respectively. In [3], the relationship of the QoS and the fairness approach was elaborated with single-antenna users. Therein, both the QoS and max-min fair problems were proven NP-hard and approximated by the semi-definite relaxation (SDR). For the multiple-antenna users, the receive beamformers are also needed in addition to the transmit ones. A common approach named alternating optimization was adopted in [4] and its related references.

However, perfect CSI at the transmitters and receivers is often unavailable and impractical due to channel estimation and quantization errors. For the cases of CSI uncertainty, robust versions of beamforming optimization have been studied. In [5], different types of imperfect CSI were considered to maximize the worst-case signal-to-interference-plus-noise ratio (SINR). Robust transceivers were designed by considering the max-min fairness problem in [6]–[8] for the MIMO interference channels. In [9], a two-tier beamforming scheme is proposed based on interference alignment to minimize the interference leakage to other cells or users for a downlink MIMO interference network. These works successfully addressed the fairness issue under different max-min or min-max forms. By contrast, efficiency issue generally gains higher interest with some QoS constraints. However, the robust QoS transceiver design based on bounded CSI uncertainty for MIMO interference channels has not been considered, to the authors' best knowledges.

In this letter, we investigate the QoS-constrained robust beamforming design with bounded CSI uncertainty in MIMO interference channels. For simplicity, only single-stream message transmission is considered. For the purpose of energy efficiency, the QoS-constrained sum power minimization problem is solved by an iterative algorithm based on the alternating optimization in a centralized manner. With fixed transmit beamformers, a closed-form solution for receiver is derived. With fixed receive beamformers, the QoS problem is recast to a semi-definite program (SDP) problem by introducing slack variables. The convergence and robustness of the proposed scheme are demonstrated by simulation, indicating potential applications in the future wireless networks such as the device-to-device communications [10], [11] and the underwater acoustic networks [12].

*Notation*: $\|\cdot\|$ denotes the spectral norm of a vector/matrix, $\|\cdot\|_F$ denotes the Frobenius norm of a matrix, $\mathrm{Tr}(\cdot)$ is the trace of a square matrix, $\mathrm{vec}(\cdot)$ is the operator that stacks up all the columns of a matrix into a vector, and $\mathcal{K} \triangleq \{1, 2, \ldots, K\}$ denotes the set of user index. The superscripts $(\cdot)^T$, $(\cdot)^*$and $(\cdot)^H$ denote the transpose, complex conjugate and Hermitian transpose, respectively. $\mathbb{R}$ and $\mathbb{C}$ denote the sets of real and complex numbers, respectively.

## 2. System Model

We consider a $K$-user MIMO interference channel denoted by $\mathrm{Tx}_k \longmapsto \mathrm{Rx}_k (k \in \mathcal{K})$, where each user is composed by one transmitter and one receiver. All the $K$ users interfere with each other. Each transmitter is equipped with $M$ antennas, while each receiver has $N$ antennas. We assume single

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data stream transmission for each user. The received signal at the $k$th receiver is

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{f}_k s_k + \sum_{i=1,i\neq k}^{K} \mathbf{H}_{ki}\mathbf{f}_i s_i + n_k, \quad i \neq k \tag{1}$$

where $\mathbf{H}_{ki} \in \mathbb{C}^{N\times M}$ is the channel matrix from $\mathrm{Tx}_i$ to $\mathrm{Rx}_k$, $\mathbf{f}_k \in \mathbb{C}^{M\times 1}$ is the transmit beamformer, $s_k$ is the transmitted symbol with $\mathbb{E}[s_k s_k*] = 1$, and $n_k \in \mathbb{C}^{N\times 1} \sim \mathcal{CN}(0, \sigma_k^2\mathbf{I})$ is the additive white Gaussian noise vector. Each transmitter is assumed to have its own power constraint, i.e., $\|\mathbf{f}_k\|^2 \leq P_k$ $(\forall k \in \mathcal{K})$.

Assuming a linear equalizer at the $k$th receiver denoted by $\mathbf{u}_k \in \mathbb{C}^{N\times 1}$, the linearly processed signal can be written as $\hat{s}_k = \mathbf{u}_k^H\mathbf{y}_k$. Then, the SINR at the $k$-th receiver is expressed as

$$\begin{aligned}\mathrm{SINR}_k &= \gamma_k(\mathbf{u}_k, \{\mathbf{f}_i\}_{i=1}^K, \{\mathbf{H}_{ki}\}_{i=1}^K)\\ &= \frac{|\mathbf{u}_k^H\mathbf{H}_{kk}\mathbf{f}_k|^2}{\sum\limits_{i=1,i\neq k}^{K} |\mathbf{u}_k^H\mathbf{H}_{ki}\mathbf{f}_i|^2 + \sigma_k^2\|\mathbf{u}_k\|^2}\end{aligned} \tag{2}$$

Under the assumption of perfect CSI, the channel $\{\mathbf{H}_{ki}\}$ is perfectly known at the transmitters and receivers. Given the required QoS $\gamma_0$, a common strategy is to minimize the total transmit power. The joint design of transceiver beamformers can be posed as

$$\begin{aligned}&\underset{\{\mathbf{u}_k\}\{\mathbf{f}_k\}}{\text{minimize}} \sum_{k=1}^{K} \|\mathbf{f}_k\|^2\\ &\quad \text{s.t.} \;\; \gamma_k(\mathbf{u}_k, \{\mathbf{f}_i\}_{i=1}^K, \{\mathbf{H}_{ki}\}_{i=1}^K) \geq \gamma_0\\ &\qquad\quad \|\mathbf{f}_k\|^2 \leq P_k\end{aligned} \tag{3}$$

*Lemma 1* [13]: Assuming $\|\mathbf{u}_k\|^2 = 1$, the optimal receive beamforming vector can be achieved by maximizing $\mathrm{SINR}_k$ (Max-SINR), i.e.,

$$\begin{aligned}\mathbf{u}_k &= \arg \max_{\|\mathbf{u}_k\|^2=1} \mathrm{SINR}_k\\ &= \frac{(\sigma_k^2\mathrm{I} + \sum_{i\neq k} \mathbf{H}_{ki}\mathbf{f}_i\mathbf{f}_i^H\mathbf{H}_{ki}^H)^{-1}\mathbf{H}_{kk}\mathbf{f}_k}{\|(\sigma_k^2\mathrm{I} + \sum_{i\neq k} \mathbf{H}_{ki}\mathbf{f}_i\mathbf{f}_i^H\mathbf{H}_{ki}^H)^{-1}\mathbf{H}_{kk}\mathbf{f}_k\|}\end{aligned} \tag{4}$$

The detailed solution of the above Max-SINR problem can be obtained by using the generalized eigen-problem and Rayleigh-Ritz theorem. We omit the detail due to space limitation.

The channel uncertainty is modeled as a bounded error [5], i.e., the CSI error matrix $\mathbf{\Delta}_{ki}$ is bounded in a certain hyper-spherical region with a radius $\epsilon_{ki}$

$$\mathcal{R} \triangleq \{\mathbf{\Delta}_{ki} : \; \|\mathbf{\Delta}_{ki}\|_F \leq \epsilon_{ki}\} \tag{5}$$

As a result, the true channel between the $k$th transmitter and the $i$th receiver can be written as

$$\mathbf{H}_{ki} = \hat{\mathbf{H}}_{ki} + \mathbf{\Delta}_{ki} \tag{6}$$

where $\hat{\mathbf{H}}_{ki} \in \mathbb{C}^{N\times M}$ is the estimated channel matrix, which is available at all transmitters and receivers. In other words, global estimated CSI is assumed.

Defining $\overline{\gamma}_k \triangleq \gamma_k(\mathbf{u}_k, \{\mathbf{f}_i\}_{i=1}^K, \{\hat{\mathbf{H}}_{ki} + \mathbf{\Delta}_{ki}\}_{i=1}^K)$ and denoting $P = [P_1, P_2, \cdots, P_K]^T$, the problem for robust transceiver design can be formulated as $\forall k, i$

$$\begin{aligned}Q(\gamma, P): \quad &\underset{\{\mathbf{u}_k\}\{\mathbf{f}_k\}}{\text{minimize}} \sum_{k=1}^{K} \|\mathbf{f}_k\|^2 &&\text{(7a)}\\ &\text{s.t.} \;\; \overline{\gamma}_k \geq \gamma &&\text{(7b)}\\ &\qquad \|\mathbf{f}_k\|^2 \leq P_k &&\text{(7c)}\\ &\qquad \|\mathbf{\Delta}_{ki}\|_F \leq \epsilon_{ki} &&\text{(7d)}\end{aligned}$$

where $\gamma > 0$ is the given QoS constraint.

In this letter, we are interested in designing robust transmit and receive beamformers to minimize the sum transmit power while the users' QoS requirements are satisfied, i.e., solving problem $Q(\gamma, P)$ in (7).

## 3. Robust Beamforming Design

We provide an iterative algorithm in this section by adopting the alternating optimization to solve the joint transceiver problem. The robust receiver beamforming vector is derived with fixed transmit beamforming vectors, while the robust transmit beamforming vector is obtained via SDR with fixed receive beamformers.

### 3.1 Robust Receive Beamforming

As the SINR expression involves independent CSI errors, the optimal receive beamforming vector can be alternatively obtained from the following problem.

$$\mathbf{u}_k = \arg \max_{\|\mathbf{u}_k\|^2=1} \min_{\{\mathbf{\Delta}_{ki}\}} \overline{\gamma}_k \tag{8}$$

Defining $\xi_k \triangleq \min_{\{\mathbf{\Delta}_{ki}\}} \overline{\gamma}_k$ as the worst-case (smallest) SINR over the uncertainty regions, from (2) we have

$$\begin{aligned}\xi_k &= \min_{\{\mathbf{\Delta}_{ki}\}} \frac{|\mathbf{u}_k^H(\hat{\mathbf{H}}_{kk} + \Delta_{kk})\mathbf{f}_k|^2}{\sum\limits_{i=1,i\neq k}^{K} |\mathbf{u}_k^H(\hat{\mathbf{H}}_{ki} + \mathbf{\Delta}_{ki})\mathbf{f}_i|^2 + \sigma_k^2\|\mathbf{u}_k\|^2}\\ &= \frac{\min_{\Delta_{kk}} |\mathbf{u}_k^H(\hat{\mathbf{H}}_{kk} + \Delta_{kk})\mathbf{f}_k|^2}{\sum_{i\neq k} \max_{\mathbf{\Delta}_{ki}} |\mathbf{u}_k^H(\hat{\mathbf{H}}_{ki} + \mathbf{\Delta}_{ki})\mathbf{f}_i|^2 + \sigma_k^2\|\mathbf{u}_k\|^2}\end{aligned} \tag{9}$$

By the triangle inequality, the numerator and denominator in (9) can be recast as

$$\begin{aligned}|\mathbf{u}_k^H(\hat{\mathbf{H}}_{kk} + \Delta_{kk})\mathbf{f}_k|^2 &= |\mathbf{u}_k^H\hat{\mathbf{H}}_{kk}\mathbf{f}_k + \mathbf{u}_k^H\Delta_{kk}\mathbf{f}_k|^2\\ &\geq |\mathbf{u}_k^H\hat{\mathbf{H}}_{kk}\mathbf{f}_k|^2 - |\mathbf{u}_k^H\Delta_{kk}\mathbf{f}_k|^2\end{aligned} \tag{10}$$

and

$$\begin{aligned}|\mathbf{u}_k^H(\hat{\mathbf{H}}_{ki} + \mathbf{\Delta}_{ki})\mathbf{f}_i|^2 &= |\mathbf{u}_k^H\hat{\mathbf{H}}_{ki}\mathbf{f}_i + \mathbf{u}_k^H\mathbf{\Delta}_{ki}\mathbf{f}_i|^2\\ &\leq |\mathbf{u}_k^H\hat{\mathbf{H}}_{ki}\mathbf{f}_i|^2 + |\mathbf{u}_k^H\mathbf{\Delta}_{ki}\mathbf{f}_i|^2\end{aligned} \tag{11}$$

respectively. Moreover, with CSI error $\|\mathbf{\Delta}_{ki}\|_F \leq \epsilon_{ki}$, $|\mathbf{u}_k^H \mathbf{\Delta}_{ki}\mathbf{f}_i|^2$ can be reformulated as

$$\begin{aligned} |\mathbf{u}_k^H \mathbf{\Delta}_{ki}\mathbf{f}_i|^2 &= \mathrm{Tr}(\mathbf{u}_k^H \mathbf{\Delta}_{ki}\mathbf{f}_i\mathbf{f}_i^H \mathbf{\Delta}_{ki}^H \mathbf{u}_k) \\ &= \mathrm{Tr}(\mathbf{u}_k\mathbf{u}_k^H \mathbf{\Delta}_{ki}\mathbf{f}_i\mathbf{f}_i^H \mathbf{\Delta}_{ki}^H) \\ &\leq \mathrm{Tr}(\mathbf{u}_k\mathbf{u}_k^H)\mathrm{Tr}(\mathbf{\Delta}_{ki}^H \mathbf{\Delta}_{ki})\mathrm{Tr}(\mathbf{f}_i\mathbf{f}_i^H) \\ &= \epsilon_{ki}^2 |\mathbf{u}_k|^2 |\mathbf{f}_i|^2 \end{aligned} \tag{12}$$

Therefore, we have

$$|\mathbf{u}_k^H (\hat{\mathbf{H}}_{kk} + \Delta_{kk})\mathbf{f}_k|^2 \geq |\mathbf{u}_k^H \hat{\mathbf{H}}_{kk}\mathbf{f}_k|^2 - \epsilon_{kk}^2 |\mathbf{u}_k|^2 |\mathbf{f}_k|^2 \tag{13}$$

and

$$|\mathbf{u}_k^H (\hat{\mathbf{H}}_{ki} + \mathbf{\Delta}_{ki})\mathbf{f}_i|^2 \leq |\mathbf{u}_k^H \hat{\mathbf{H}}_{ki}\mathbf{f}_i|^2 + \epsilon_{ki}^2 |\mathbf{u}_k|^2 |\mathbf{f}_i|^2 \tag{14}$$

As a result, $\xi_k$ in (9) can be expressed as

$$\xi_k = \frac{|\mathbf{u}_k^H \hat{\mathbf{H}}_{kk}\mathbf{f}_k|^2 - \epsilon_{kk}^2 |\mathbf{u}_k|^2 |\mathbf{f}_k|^2}{\sum_{i=1, i\neq k}^{K} |\mathbf{u}_k^H \hat{\mathbf{H}}_{ki}\mathbf{f}_i|^2 + \epsilon_{ki}^2 |\mathbf{u}_k|^2 |\mathbf{f}_i|^2 + \sigma_k^2 \|\mathbf{u}_k\|^2} \tag{15}$$

Therefore, the beamforming vector at the $k$-th receiver is recast as

$$\mathbf{u}_k = \arg \max_{\|\mathbf{u}_k\|^2 = 1} \xi_k \tag{16}$$

Define

$$\mathbf{A}_k = \hat{\mathbf{H}}_{kk}\mathbf{f}_k\mathbf{f}_k^H \hat{\mathbf{H}}_{kk}^H - \epsilon_{kk}^2 \|\mathbf{f}_k\|^2 \mathbf{I}_N \tag{17}$$

and

$$\mathbf{B}_k = \sum_{i=1, i\neq k}^{K} (\hat{\mathbf{H}}_{ki}\mathbf{f}_i\mathbf{f}_i^H \hat{\mathbf{H}}_{ki}^H + \epsilon_{ki}^2 \|\mathbf{f}_i\|^2 \mathbf{I}_N) + \sigma_k^2 \mathbf{I}_N \tag{18}$$

The robust receive beamforming vector is given as the principle eigenvector corresponding to the largest generalized eigenvalue of $\mathbf{B}_k^{-1}\mathbf{A}_k$ according to Lemma 1 in (4).

### 3.2 Robust Transmit Beamforming

Since the SINR constraint in (7b) is not convex, it can be rewritten as

$$\frac{1}{\gamma} |\mathbf{u}_k^H (\hat{\mathbf{H}}_{kk} + \Delta_{kk})\mathbf{f}_k|^2 \geq \sum_{i=1, i\neq k}^{K} |\mathbf{u}_k^H (\hat{\mathbf{H}}_{ki} + \mathbf{\Delta}_{ki})\mathbf{f}_i|^2 + \|\mathbf{u}_k\|^2 \sigma_k^2 \tag{19}$$

Introducing auxiliary variables $\{b_{ki}\} \in \mathbb{R}^+$, the SINR constraints (7b) and (7d) can be recast as $\forall i \neq k$

$$|\mathbf{u}_k^H (\hat{\mathbf{H}}_{kk} + \Delta_{kk})\mathbf{f}_k| \geq t_k \tag{20}$$

$$|\mathbf{u}_k^H (\hat{\mathbf{H}}_{ki} + \mathbf{\Delta}_{ki})\mathbf{f}_i| \leq b_{ki} \tag{21}$$

$$\forall \|\mathbf{\Delta}_{ki}\|_F \leq \epsilon_{ki} \tag{22}$$

where $t_k \triangleq \sqrt{\gamma\Big(\sum_{i\neq k} b_{ki}^2 + \|\mathbf{u}_k\|^2 \sigma_k^2\Big)}$.

Using the properties of Kronecker products [14], the above inequalities can be equivalently reformulated as $\forall i \neq k$

$$|(\mathbf{f}_k^T \otimes \mathbf{u}_k^H)\mathrm{vec}(\hat{\mathbf{H}}_{kk} + \Delta_{kk})| \geq t_k \tag{23}$$

$$|(\mathbf{f}_i^T \otimes \mathbf{u}_k^H)\mathrm{vec}(\hat{\mathbf{H}}_{ki} + \mathbf{\Delta}_{ki})| \leq b_{ki} \tag{24}$$

$$\forall \|\mathrm{vec}(\mathbf{\Delta}_{ki})\| \leq \epsilon_{ki} \tag{25}$$

Let $\mathbf{w}_{ki} = \mathbf{f}_i^T \otimes \mathbf{u}_k^H$, $\mathbf{h}_{ki} = \mathrm{vec}(\hat{\mathbf{H}}_{ki})$, and $\mathbf{e}_{ki} = \mathrm{vec}(\mathbf{\Delta}_{ki})$. We can further rewrite the above constraints as $\forall i \neq k$

$$(\mathbf{h}_{kk} + \mathbf{e}_{kk})^H \mathbf{w}_{kk}^H \mathbf{w}_{kk}(\mathbf{h}_{kk} + \mathbf{e}_{kk}) \geq t_k^2 \tag{26}$$

$$(\mathbf{h}_{ki} + \mathbf{e}_{ki})^H \mathbf{w}_{ki}^H \mathbf{w}_{ki}(\mathbf{h}_{ki} + \mathbf{e}_{ki}) \leq b_{ki}^2 \tag{27}$$

$$\forall \|\mathbf{e}_{ki}\| \leq \epsilon_{ki} \tag{28}$$

Defining $\mathbf{F}_i \triangleq \mathbf{f}_i\mathbf{f}_i^H$, $\mathbf{R}_k \triangleq \mathbf{u}_k\mathbf{u}_k^H$, and $\mathbf{W}_{ki} \triangleq \mathbf{w}_{ki}^H\mathbf{w}_{ki}$, we have $\mathbf{W}_{ki} = \mathbf{F}_i^T \otimes \mathbf{R}_k$ with the properties of Kronecker products, where $\mathrm{rank}(\mathbf{F}_i) = 1$. Then the above non-convex constraints can be relaxed using the SDR or S-procedure as follows.

*Lemma 2*: (S-procedure [15]) Let $\phi_i(\mathbf{x}) \triangleq \mathbf{x}^H A_i \mathbf{x} + \mathbf{b}_i^H \mathbf{x} + \mathbf{x}^H \mathbf{b}_i + c_i$, for $i = 1, 2$, where $A_i \in \mathbb{C}^{N_t \times N_t}$ is a complex Hermitian matrix, $\mathbf{b}_i \in \mathbb{C}^{N_t}$ and $c_i \in \mathbb{R}$. Suppose there exists an $\bar{\mathbf{x}} \in \mathbb{C}^{N_t}$ with $\phi_1(\bar{\mathbf{x}}) < 0$. Then the two conditions are equivalent:

1) $\phi_2(\bar{\mathbf{x}}) \geq 0$ for all $\mathbf{x}$ satisfying $\phi_1(\bar{\mathbf{x}}) \leq 0$;
2) There exists a $\lambda \geq 0$ such that

$$\begin{bmatrix} A_2 & \mathbf{b}_2 \\ \mathbf{b}_2^H & c_2 \end{bmatrix} + \lambda \begin{bmatrix} A_1 & \mathbf{b}_1 \\ \mathbf{b}_1^H & c_1 \end{bmatrix} \succeq 0$$

By applying the above S-procedure, we can recast (26)–(28) as the linear matrix inequalities, $\forall k, i$

$$T_{kk}(\mathbf{F}_k, \{b_{ki}\}_{i\neq k}, \lambda_{kk}) \triangleq \begin{bmatrix} \mathbf{W}_{kk} + \lambda_{kk}\mathbf{I} & \mathbf{W}_{kk}\mathbf{h}_{kk} \\ \mathbf{h}_{kk}^H \mathbf{W}_{kk} & \mathbf{h}_{kk}^H \mathbf{W}_{kk}\mathbf{h}_{kk} - t_k^2 - \lambda_{kk}\epsilon_{kk}^2 \end{bmatrix} \succeq 0 \tag{29}$$

$$\Phi_{ki}(\mathbf{F}_i, b_{ki}, \lambda_{ki}) \triangleq \begin{bmatrix} -\mathbf{W}_{ki} + \lambda_{ki}\mathbf{I} & -\mathbf{W}_{ki}\mathbf{h}_{ki} \\ -\mathbf{h}_{ki}^H \mathbf{W}_{ki} & b_{ki}^2 - \mathbf{h}_{ki}^H \mathbf{W}_{ki}\mathbf{h}_{ki} - \lambda_{ki}\epsilon_{ki}^2 \end{bmatrix} \succeq 0 \tag{30}$$

$$\lambda_{ki} \geq 0 \tag{31}$$

where $\{\lambda_{ki} \geq 0\}$ are slack variables. Consequently, (7) is reformulated as a SDP form

$$\begin{aligned} \underset{\{\mathbf{F}_k\},\{b_{ki}\},\{\lambda_{ki}\}}{\text{minimize}} \quad & \sum_{k=1}^{K} \mathrm{Tr}(\mathbf{F}_k) \\ \text{s.t.} \quad & T_{kk}(\mathbf{F}_k, \{b_{ki}\}_{i\neq k}, \lambda_{kk}) \succeq 0 \\ & \Phi_{ki}(\mathbf{F}_i, b_{ki}, \lambda_{ki}) \succeq 0, \ \forall i \neq k \\ & \mathbf{F}_k \succeq 0, \mathrm{Tr}(\mathbf{F}_k) \leq P_k \end{aligned} \tag{32}$$

As previously noted, rank-one solutions of (32) are considered feasible. If the obtained solution is not of rank-one, then additional solution approximation procedure, such as the Gaussian randomization method [16] can be employed to generate a set of rank-one solutions to (32). In general, a good approximation solution can be obtained by sampling enough time from the complex-valued Gaussian distribution, i.e., $\mathbf{f}_k \sim \mathcal{CN}(0, \mathbf{F}_k)$. However, we find that the maximum eigenvalue related eigenvectors of $\mathbf{F}_k$ can be chosen as the approximate solutions $\mathbf{f}_k$ in simulations, which greatly simplifies the solving processing.

In summary, the proposed algorithm is outlined as Algorithm 1.

**Algorithm 1** The Proposed Algorithm

**Initialize:** Initialize $\{\mathbf{f}_k^{(0)}\}$;
1: **for** $\{\mathbf{u}_k^{(n)}\}$, $\{\mathbf{f}_k^{(n)}\}$ ($n \geq 1$ is the number of iterations) **do**
2: Substitute $\{\mathbf{f}_k^{(n-1)}\}$ into (15);
3: Solve problem (16) to get $\{\mathbf{u}_k^{(n)}\}$;
4: Substitute $\{\mathbf{u}_k^{(n)}\}$ into (32);
5: Solve problem (32) to get $\{\mathbf{f}_k^{(n)}\}$;
6: **end for**

Since each node performs the same algorithm with global CSI, the proposed scheme is working in a centralized matter. The computational overhead is mainly caused by solving the SDP problem which performs $O((M^2 + N^2)K^{3.5}M^{4.5}N^{4.5})$ arithmetic operations in each iteration [15]. Let the iteration number be $L$. Since each node runs the same algorithm, the total complexity is $2KL$-fold. Obviously, this is a polynomial complexity. We should remark that the initialization of $\{\mathbf{f}_k^0\}$ should be the same choice at all nodes. At the same time, all nodes should agree on the iteration number. Otherwise, the mismatch problem might degrade the system performance. The convergence of the proposed scheme will be shown in the next section.

## 4. Simulation Results

In this section, we provide numerical example to evaluate the performance of the proposed algorithm. We consider a three-user ($K = 3$) MIMO interference channel with $M = 4$ antennas at each transmitter and $N = 2$ antennas at each receiver. For all simulations, the MIMO channel is randomly generated from independent and identically distributed (i.i.d.) complex Gaussian distribution with zero-mean and unit-variance. 1000 channel realization are considered. The noise power is set to $\sigma_k^2 = \sigma^2 = 1$.

Since there is no related reference, we consider the perfect CSI scenario, i.e., $\epsilon_{ki} = 0$ ($\forall k, i$) as the baseline to assess the performance of the proposed scheme. To solve the optimization problems we use CVX, a package for solving convex programs [17].

Figure 1 demonstrates the convergence behavior of the proposed algorithm. It can be seen that the total transmit power values decrease monotonically as expected, and most of the improvements are achieved in the first few iterations.

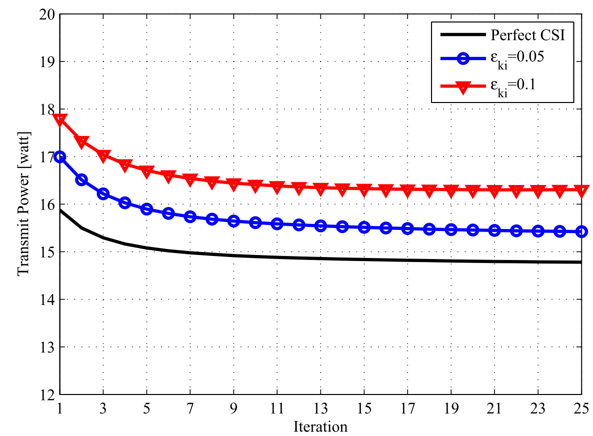


**Fig. 1** Convergence behavior of the proposed algorithm.

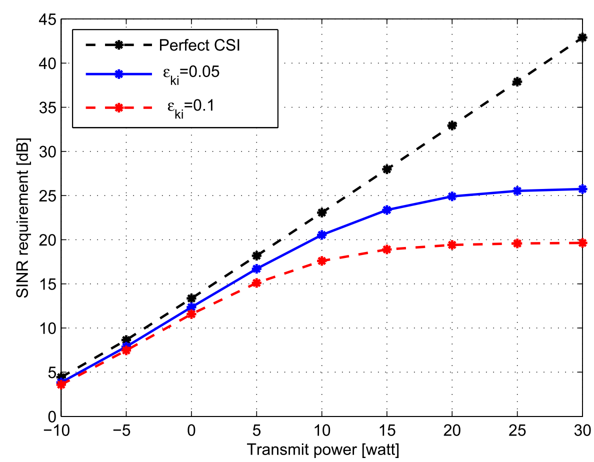


**Fig. 2** Average target SINR versus the average transmitted power.

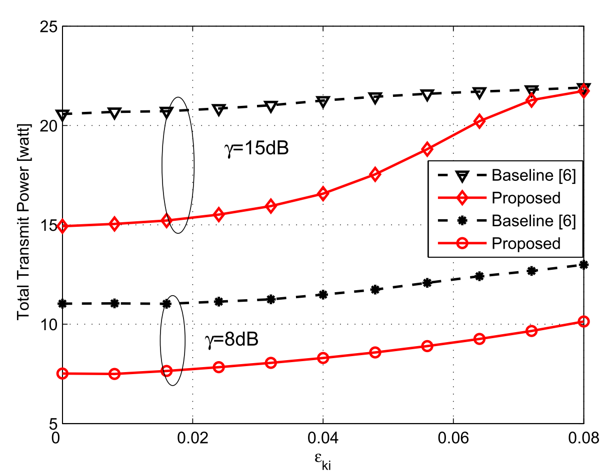


**Fig. 3** Total transmit power versus CSI errors.

In the following simulation, we set the iteration number as 6, which is quite close to the converging value of target transmit power. In this case with $K = 3$ and $L = 6$, the SDP problem will be solved 36 times for each channel instance.

Figure 2 shows the equal SINR requirement of each user versus the average transmitted power. As expected, the achievable SINR increases with the growth of the average transmitted power. This figure also illustrates the negative effect of the channel uncertainty imposed on the growth of the target SINR. The higher uncertainty, the lower achievable target SINR.

In Fig. 3, we compare the total transmit power performance of the proposed and baseline scheme in [6], where the minimum SINR requirements of each user are 8 dB and 15 dB, respectively. It can be observed that the total transmit power increases with increasing CSI error, and the proposed scheme needs lower total transmit power. For example, the proposed algorithm saves 3.2 dBm transmit power compared to the baseline for $\gamma = 8$ dB with $\epsilon_{ki} = 0.1$.

## 5. Conclusion

Robust linear transceiver design for MIMO interference channels with QoS constraints was provided under bounded CSI error model. The problem of the total power minimization with QoS constraints is formulated and solved. The robust beamformers at the receiver are achieved as modified max-SINR, while the robust transmit beamformers are obtained through SDP. This alternating approach determines a centralized algorithm for the transceiver design. Simulations show the fast convergence and the good robustness of the proposed scheme, which provides potential for its practical applications.

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