SUMMARY  This paper proposes a new design criterion of adaptively scaled belief (ASB) in Gaussian belief propagation (GaBP) for large multi-user multi-input multi-output (MU-MIMO) detection. In practical MU detection (MUD) scenarios, the most vital issue for improving the convergence property of GaBP iterative detection is how to deal with belief outliers in each iteration. Such outliers are caused by modeling errors due to the fact that the law of large number does not work well when it is difficult to satisfy the large system limit. One of the simplest ways to mitigate the harmful impact of outliers is belief scaling. A typical approach for determining the scaling parameter for the belief is to create a look-up table (LUT) based on the received signal-to-noise ratio (SNR) through computer simulations. However, the instantaneous SNR differs among beliefs because the MIMO channels in the MUD problem are random; hence, the creation of LUT is infeasible. To stabilize the dynamics of the random MIMO channels, we propose a new transmission block based criterion that adapts belief scaling to the instantaneous channel state. Finally, we verify the validity of ASB in terms of the suppression of the bit error rate (BER) floor.

key words: multi-user multi-input multi-output (MU-MIMO), Gaussian belief propagation (GaBP), iterative detection, soft interference cancellation, adaptive belief scaling

1. Introduction

Large multi-user multi-input multi-output (MU-MIMO) systems for achieving higher spectral efficiency have emerged as a key technology in wireless communications [1], [2]. Large MIMO systems can increase data rate with the aid of spatial multiplexing gain and improve detection reliability thanks to the spatial diversity gain [2]. In principle, these beneficial gains are enhanced with larger dimensions of MIMO channels, which are constructed by plural transmit (TX) and receive (RX) antennas in rich scattering environments. However, signal detection in the uplink of large MIMO systems for achieving higher spectral efficiency has been a significant issue for improving the convergence property of GaBP iterative detection is how to deal with belief outliers in each iteration. Such outliers are caused by modeling errors due to the fact that the law of large number does not work well when it is difficult to satisfy the large system limit. One of the simplest ways to mitigate the harmful impact of outliers is belief scaling. A typical approach for determining the scaling parameter for the belief is to create a look-up table (LUT) based on the received signal-to-noise ratio (SNR) through computer simulations. However, the instantaneous SNR differs among beliefs because the MIMO channels in the MUD problem are random; hence, the creation of LUT is infeasible. To stabilize the dynamics of the random MIMO channels, we propose a new transmission block based criterion that adapts belief scaling to the instantaneous channel state. Finally, we verify the validity of ASB in terms of the suppression of the bit error rate (BER) floor.

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1. Introduction

Large multi-user multi-input multi-output (MU-MIMO) systems for achieving higher spectral efficiency have emerged as a key technology in wireless communications [1], [2]. Large MIMO systems can increase data rate with the aid of spatial multiplexing gain and improve detection reliability thanks to the spatial diversity gain [2]. In principle, these beneficial gains are enhanced with larger dimensions of MIMO channels, which are constructed by plural transmit (TX) and receive (RX) antennas in rich scattering environments. However, signal detection in the uplink of large MIMO involves a large-scale multi-user detection (MUD) problem, which is computationally expensive [3]. Maximum likelihood detection (MLD) is universally recognized as the most powerful nonlinear MUD. However, its computational complexity increases exponentially with the number of multiplication operations. Therefore, many researchers have attempted to develop novel schemes for realizing detectors with low computational complexity without sacrificing their detection capability significantly, e.g., sphere detector (SD) [4]. However, the complexity remains high in large-scale MUD.

A breakthrough of large-scale MUD is to construct message passing (MP) algorithm that was Bayes-optimal in the large system limit, where the input and output dimensions $M$ and $N$ increase to infinity while the compression rate $\xi = N/M$ is kept constant. A variety of MP algorithms based on belief propagation (BP) have been proposed in a wide range of fields, and the BP-based algorithm was conjectured to achieve the optimal performance under a specific condition at the large system limit [5]. Therefore, a rigorous proof of the convergence property of the BP-based algorithm has attracted more and more attention. As a specific condition, the BP-based algorithm was numerically shown to achieve Bayes-optimal performance when the compression rate is larger than the so-called BP threshold [6]. To give a rigorous proof, approximated message passing (AMP) was proposed in [7] and proved in [8] to achieve optimal performance for independent and identically distribution (i.i.d.) Gaussian measurements. Even when the i.i.d. assumption of measurement matrices is not satisfied, expectation propagation (EP) based algorithm [9], [10], which is systematically derived from Minka’s EP framework [11], was proved in [12], [13] to achieve optimal performance under heuristic assumptions. However, the EP-based algorithm requires matrix inversion in each iteration process, and this operation remains undesirable in the large-scale MUD. To reduce the computational complexity by removing matrix inversion, Gaussian BP (GaBP) was presented in [14], and the convergence analysis and proposal to improve the convergence property were also presented in [15]. These studies are undoubtedly vital to understand the behavior of BP-based algorithms.

However, it is difficult to satisfy the large system limit in practical scenarios, for example, the large-scale MUD addressed in this paper. In practical large-scale MUD scenarios, a sufficient number of RX antennas may be unavailable because of the limits on the size, weight, cost and/or power consumption of the receiver. Moreover, the system is subject to spatial fading correlations. As a result, the large system limit is not always satisfied [16]–[18]. Then, the convergence property of the BP-based algorithm for MUD is significantly deteriorated, and deviates from the limit property presented in the above literature. As is well known, unfortunately, it is impractical to analyze the BP-based algorithm directly without the assumption of the large system limit. To the best of our knowledge, the convergence analysis without the
assumption remains open issues. A major focus in this paper is, therefore, to improve the detection capability of the BP-based algorithm when it is difficult to satisfy the large system limit.

When the large system limit is not satisfied, the dominant factor of ill convergence behavior of the BP based algorithm is the harmful effects of approximation errors due to a fact that the law of large number does not work well. To improve the convergence property by mitigating the negative effects of the approximation errors, several methodologies have been proposed. Damped belief [14], [19] is widely used for alleviating ill convergence behavior of the BP-based algorithm. However, it is not effective in correlated MIMO channels. The node selection (NS) method [20] has been proposed to suppress the impairment of spatial fading correlation. The NS method can be naturally regarded as an extension of damped belief. However, even if we employ these conventional techniques for improving the convergence property, GaBP is still subject to ill convergence behavior when the large system limit is not satisfied.

To further improve the convergence property of GaBP, we introduce a belief scaling, which is inspired by the ideas presented in [21], [22]. To reduce the computational complexity of the turbo decoder, the maximum log maximum a-posterior probability (Max-Log-MAP) algorithm is often used instead of the Log-MAP algorithm. However, the maximum approximation deteriorates the reliability of log likelihood ratio (LLR) belief. The scaling operation is applied to the approximated LLR to compensate the approximation errors. The appropriate scaling parameter is typically determined by computer simulations for each received signal-to-noise ratio (SNR). From the perspective of the analogy between approximation errors in turbo decoding and in GaBP for large-scale MUD, it is worth considering appropriate scaling parameters of GaBP. In this case, the scaling parameter relies not only on the received SNR but also on the channel matrix $H \in \mathbb{C}^{N \times M}$. Therefore, it is infeasible to find an optimal scaling parameter for each channel matrix $H$ and SNR in advance.

This paper makes three major contributions as follows:\footnote{This paper extends our conference papers [23], [24] by including: simulation-based optimization for the scaling parameter, techniques for dealing with highly correlated MIMO channels caused by a square array of equally spaced receive antennas, and a detailed complexity analysis.}

- Adaptively scaled belief (ASB) is proposed to improve the convergence property of GaBP by dealing with the approximation errors as belief outlier (from the ideal distribution in the large system limit) when the large system limit is not satisfied.
- To reduce the computational complexity of GaBP, matched filter (MF) belief is defined instead of the traditional log likelihood ratio (LLR) belief.
- Further reduction of the computational complexity is potentially possible due to a fact that the noise variance estimator is not required for MF belief with ASB.

The remainder of this paper is organized as follows. Section 2 presents a mathematical signal model, and clarifies the approximation errors when the large system limit is not satisfied as a preliminary step. Section 3 defines the simple MF belief for reducing the computational complexity. Section 4 presents the novel ASB that attains excellent convergence behavior in GaBP. Section 5 describes the validation of the proposed method through computer simulations. Finally, Sect. 6 concludes the paper with a brief summary.

2. Preliminaries

2.1 Mathematical Notation

Throughout this paper, vectors and matrices are denoted by lower-case and upper-case bold-face letters, respectively. Furthermore, $\cdot^*$, $\cdot^T$, and $\cdot^H$ denote conjugate, transpose, and conjugate transpose, respectively. $P_{A|B}[a|b]$ and $P_{A|B}(a|b)$ respectively denote the conditional probability mass function (PMF) and probability density function (PDF) of a realization $a$ for random variable $A$ given the occurrence of a realization $b$ for random variable $B$. $\mathbb{E}_A[\cdot]$ denotes the expectation of random variable $A$. $\mathbb{E}_{A|B=b}[\cdot]$ denotes the conditional expectation of random variable $A$ given the occurrence of a realization $b$ for random variable $B$. $\Re[\cdot]$ and $\Im[\cdot]$ denote the real and imaginary parts of the complex values, respectively. $\mathbb{C}^{a \times b}$ denotes a complex field of size $a \times b$. $I_a$ represents an $a \times a$ square identity matrix. diag $[\cdot]$ denotes a diagonal matrix with the elements of $\cdot$.

2.2 System Model

In this paper, we consider uplink MUD, where the BS has $N$ receive (RX) antennas and $M$ individual user equipments (UEs) have a single TX antenna. The $m$-th UE conveys a modulated symbol $x_m$, which represents one of $Q$ constellation points $X = \{x_1, \ldots, x_q, \ldots, x_Q\}$. The average energy of the constellations in the set $X$ is denoted by $E_x$. Let $x = [x_1, \ldots, x_m, \ldots, x_M]^T \in \mathbb{C}^{M \times 1}$ and $y = [y_1, \ldots, y_m, \ldots, y_N]^T \in \mathbb{C}^{N \times 1}$ denote the TX and RX symbol vectors, respectively. Assuming frequency flat and slow fading environments, the RX vector $y$ is given by

$$y = Hx + z. \quad (1)$$

$H \in \mathbb{C}^{N \times M}$ is an $N \times M$ channel matrix. The $n$-th entry of $y$ is expressed as

$$y_n = h_{n}^T x + z_n, \quad (2)$$

where the $n$-th row vector of $H$ is denoted by $h_{n}^T$. The vector $z = [z_1, \ldots, z_m, \ldots, z_N]^T \in \mathbb{C}^{N \times 1}$ is a complex additive white Gaussian noise (AWGN) vector, whose entries $z_n$ obey independent identically distributed (i.i.d.) complex Gaussian distribution with zero mean and $N_0$ variance $CN(0, N_0)$, where $N_0$ is the noise spectral density and the covariance matrix is given by $\mathbb{E}_Z[zz^H] = N_0 I_N$. With the above-mentioned signal model, the conditional PDF of RX vector...
Throughout the whole paper, we assume Gray-coded quadrature amplitude modulation (QAM) and assume a large system limit as derived by the expectation of random variable $Z$ and $\bar{X}_m$ as

$$
\psi_{n,m} = E_Z, \bar{X}_m \left\{ v_{n,m} \right\} = E_{\bar{X}_m} \left\{ h^*_n x_m \bar{X}_m h^*_n \right\} + N_0
$$

With the assistance of $h_n$ and $\psi_{n,m}$ under SGA, the PDF of $\tilde{y}_{n,m}$ is expressed as

$$
p_{\tilde{y}_{n,m}}(\tilde{y}_{n,m} | x_m) = \frac{1}{\pi \psi_{n,m}} \exp \left[ -\frac{|\tilde{y}_{n,m} - h_n x_m|^2}{\psi_{n,m}} \right].
$$

On the basis of (8), an LLR $\alpha_{n,m}$ is defined by

$$
\alpha_{n,m} = \gamma_{n,m} \bar{h}_n \tilde{y}_{n,m}
$$

where we have

$$
\gamma_{n,m} = \frac{4c}{\psi_{n,m}} = \frac{2\sqrt{E_s}}{N_0 + E_n h^*_n h_n^H}.
$$

Further, (9) implies that $\alpha_{n,m}$ is generated by $\gamma_{n,m}$ scaling followed by MF (MF & Scaling), as shown in Fig. 1.

When the large system limit is satisfied, $\alpha_{1,m}, \ldots, \alpha_{N,m}$ are not correlated owing to SGA. Then, a joint LLR corresponding to TX symbol $x_m$ is simply obtained by

$$
\beta_m = \sum_{n=1}^{N} \alpha_{n,m}.
$$

If the belief propagation is given by the joint LLR $\beta_m$ of (11), GaBP is subject to ill convergence behavior of iterative detection owing to the correlation between $y_n$ and $\alpha_{n,m}$ included in $\beta_m$ at the next iteration process. Therefore, in the GaBP regime, the prior belief is typically provided by an extrinsic LLR $\lambda_{n,m}$, which is given by

$$
\beta_m = \beta_m - \alpha_{n,m} = \sum_{i=1, i\neq m}^{N} \alpha_{i,m}.
$$

Finally, in the traditional GaBP, $\beta_{n,m}$ is fed back to the soft symbol generator (soft SG) as the prior belief $\lambda_{n,m}$. Here, we have a prior belief vector $\lambda_n = [\lambda_{1,n}, \ldots, \lambda_{N,n}, \ldots, \lambda_{N,M}]^\top$ by using (12). When the occurrence of the vector $x$ is equiprobable ($P_X[x] = \frac{1}{2^M}$, $\forall x \in \mathbb{C}^M$)
After updating the LLR $\alpha_{n,m}$ (11), (12), and (14) are performed again in each iteration.

When the number of iterations reaches the predetermined maximum number of iterations, $x_m$ is detected by

$$\hat{x}_m = \sqrt{\frac{E_s}{2}} \left( \text{sgn} \{ \Re \{ \beta_m \} \} + j \text{sgn} \{ \Im \{ \beta_m \} \} \right),$$

where $\text{sgn}[\cdot]$ denotes the operation for extracting the sign of a number.

### 2.4 Issues of GaBP in Practical Scenarios

In principle, the GaBP algorithm works on the assumption that the large system limit is satisfied or approximately satisfied. However, it is difficult to satisfy the assumption in practical large-scale MUD scenarios according to the circumstances. When the accuracy of SGA in (16) deteriorates, the second term in (15): $h_n^T (\bar{x}_m - \tilde{x}_m)$ is fluctuated based on the instantaneous channel matrix $\bm{H}$ and TX vector $\mathbf{x}$. Then, $\alpha_1, \ldots, \alpha_m$ are correlated owing to the off-diagonal elements of a Gram matrix $\bm{H}^H \bm{H}$, especially in correlated MIMO channels. As a result, the approximation errors appear as abnormal noise enhancements in the prior belief $\lambda_{n,m} = \beta_{n,m}$ in (12). Therefore, the simplified conditional expectation in (14) includes errors from the exact soft symbol vector of (13). Consequently, GaBP is subject to ill convergence behavior of iterative detection due to the noise enhancements in each iteration, resulting in the error floor of the bit error rate (BER) performance.

In addition, the instantaneous variables $\bm{H}$ and $\mathbf{x}$ are no longer the average characteristic. Therefore, it is quite difficult to analyze the convergence property of GaBP iterative detection when the large system limit is not satisfied.

### 2.5 Conventional Techniques

In uncorrelated MIMO channels, the damped belief is an effective technique [14], [19] to suppress the harmful effect of abnormal noise enhancements by using a weighted average of the previous belief (i.e., prior belief in the previous iteration) and the current belief (i.e., prior belief in the current iteration) as the prior belief. However, in correlated MIMO channels, the damped belief is not sufficiently effective. To suppress the impairment of spatial fading correlation on the BS side, the node selection (NS) method is applied to (12), which is inspired by the idea presented in [20]. The NS method is discussed in detail in Sect. 3.2.

### 3. MF Belief for Reducing the Computational Complexity

#### 3.1 MF Belief in GaBP (MF-GaBP)

In this section, a simple MF belief is defined to reduce the computational complexity. As aforementioned, the GaBP algorithm suffers from the approximation errors when the
large system limit is not tightly satisfied. Then, the approximated LLR including errors may no longer have advantages commensurate with its high computational burden. Here, we suggest that the LLR weight \(\gamma_{n,m} \) in (20) is replaced with an arbitrary weight \(\gamma\). Consequently, (20) is replaced by

\[
\alpha_{n,m} = \gamma h_{n,m}^* \tilde{y}_{n,m}, \tag{23}
\]

where \(\gamma\) is a constant value whereas \(\gamma_{n,m}\) in (21) is a non-constant value. In this case, the variable \(\alpha_{n,m}\) is no longer the LLR belief but the scaled MF belief. Therefore, we are not able to calculate the conditional expectation of (13) using the soft symbol generator of (14). To obtain the soft symbol \(\tilde{x}_{n,m}\) from the MF belief, some appropriate mathematical manipulations from (14) are required.

According to (13), to compute the exact conditional expectation, the PDF \(p_{\lambda_n|x}(\lambda_n|x)\) is essential. In the case of the first iteration \((k = 1, \tilde{x}_n = 0)\), from (2), (5), (12) and (23), the extrinsic scaled MF belief is given by

\[
\beta_{n,m} = \sum_{i=1}^{N} \alpha_{i,m} = \gamma \sum_{i=1}^{N} h_{i,n}^* h_{i,m} x + \sum_{i=1}^{N} h_{i,n}^* z_i. \tag{24}
\]

Then, the prior belief vector \(\lambda_n\) is expressed as

\[
\lambda_n = [\beta_{n,1}, \ldots, \beta_{n,m}, \ldots, \beta_{n,M}]^T \gamma \sum_{i=1}^{N} h_{i,n}^* h_{i,m} x + \sum_{i=1}^{N} h_{i,n}^* z_i. \tag{25}
\]

Assuming that the prior belief vector \(\lambda_n\) in (25) obeys a multivariate Gaussian distribution, the statistical behavior of \(\lambda_n\) is modeled as

\[
\lambda_n = \Theta_n x + \phi_n, \tag{26}
\]

where we define

\[
\Theta_n = \gamma \sum_{i=1}^{N} h_{i,n}^* h_{i,n}^*, \quad \phi_n = \gamma \sum_{i=1}^{N} h_{i,n}^* z_i. \tag{27}
\]

\(\Theta_n\) and \(\phi_n\) are the effective gain matrix and noise vector, respectively. Then, the covariance matrix \(\Omega_n\) of \(\phi_n\) is given by

\[
\Omega_n = \mathbb{E} z \left[ \phi_n^H \phi_n \right] = \gamma^2 N_0 \sum_{i=1}^{N} h_{i,n}^* h_{i,n}^* = \gamma N_0 \Theta_n. \tag{28}
\]

With the assistance of \(\Theta_n\) and \(\Omega_n\) under the Gaussian approximation, the conventional PDF of \(\lambda_n\) is expressed as

\[
p_{\lambda_n|x}(\lambda_n|x) = \frac{1}{\pi^{N/2} \Omega_n^{1/2}} \exp \left[ - (\lambda_n - \Theta_n x)^H \Omega_n^{-1} (\lambda_n - \Theta_n x) \right]
\]

\[
\propto \exp \left[ 2 \Re \left\{ \lambda_n^H \Xi_n x - x^H \Phi_n x \right\} \right], \tag{29}
\]

where we have

\[
\Xi_n = \Omega_n^{-1} \Theta_n = (\gamma N_0 \Theta_n)^{-1} \Theta_n = \frac{1}{\gamma N_0} I_M. \tag{30}
\]

\[
\Phi_n = \Theta_n^H \Omega_n^{-1} \Theta_n = \Theta_n^H (\gamma N_0 \Theta_n)^{-1} \Theta_n = \frac{1}{\gamma N_0} \Theta_n. \tag{31}
\]

where \(\Theta_n^H = \Theta_n\) because it is a Hermitian matrix.

Here, we should focus on the stochastic property of MIMO channels as the output dimension \(N\) increases. More specifically, the Gram matrix \(HH^H\) approximately approaches a diagonal matrix if the size of \(H\) is sufficiently large. Such stochastic behavior is referred to as channel hardening effect [3]. Thus, under the condition of the large system limit, \(\Theta_n = \gamma \left[ HH^H - h_{n,m}^H h_{n,m}^* \right]\) can be approximately regarded as a diagonal matrix as

\[
\lim_{N \to \infty} \Theta_n = \lim_{N \to \infty} \gamma \left[ HH^H - h_{n,m}^H h_{n,m}^* \right] \approx (N-1)I_M. \tag{32}
\]

Note that we need to accept the slight deterioration in the BER performance due to this approximation when the scaled MF belief is utilized instead of the LLR belief. With the approximation of (32), we have diagonal matrices

\[
\Xi_n = \frac{1}{\gamma N_0} I_M, \quad \Phi_n \approx \frac{N-1}{N_0} I_M. \tag{33}
\]

When \(x_n\) assumes PSK signaling and \(\Phi_n\) is a diagonal matrix, \(x^H \Phi_n x\) has a constant value that does not rely on \(x\). Eventually, (13) is expressed as

\[
\tilde{x}_n = \frac{\sum_{x \in \mathcal{X}^{M \times 1}} x \exp \left[ \frac{2}{\gamma N_0} \Re \left\{ A_n^H x \right\} \right]}{\sum_{x \in \mathcal{X}^{M \times 1}} \exp \left[ \frac{2}{\gamma N_0} \Re \left\{ A_n^H x'^\dagger \right\} \right]}. \tag{34}
\]

For QPSK signaling, after some manipulations, each soft symbol is simply given by

\[
\tilde{x}_{n,m} = c \left( \tanh \left( \sqrt{\frac{E_s}{2}} \Re \left\{ A_n^H \right\} \right) + j \tanh \left( \sqrt{\frac{E_s}{2}} \Im \left\{ A_n^H \right\} \right) \right), \tag{35}
\]

where we have

\[
\lambda_{n,m} = \beta_{n,m} = \gamma \sum_{i=1}^{N} h_{i,m}^* \tilde{y}_{i,m}. \tag{36}
\]

Substituting (36) into (35), \(\tilde{x}_{n,m}\) is obtained by

\[
\tilde{x}_{n,m} = c \left( \tanh \left( \sqrt{\frac{E_s}{2}} \Re \left\{ \gamma \sum_{i=1}^{N} h_{i,m}^* \tilde{y}_{i,m} \right\} \right) + j \tanh \left( \sqrt{\frac{E_s}{2}} \Im \left\{ \gamma \sum_{i=1}^{N} h_{i,m}^* \tilde{y}_{i,m} \right\} \right) \right)
\]

\[
= \sqrt{\frac{E_s}{2}} \left( \tanh \left( \Re \left\{ A_{n,m} \right\} \right) + j \tanh \left( \Im \left\{ A_{n,m} \right\} \right) \right), \tag{37}
\]

where we define

\[
A_{n,m} = \frac{\sqrt{E_s}}{N_0} \sum_{i=1}^{N} h_{i,m}^* \tilde{y}_{i,m}. \tag{38}
\]
From (37) and (38), we may put $\gamma = 1$ because the contribution of $\gamma$ is disappeared. Therefore, the scaled MF belief in (23) is simply rewritten as MF belief:
\[ \alpha_{n,m} = h'_{n,m} \tilde{y}_{n,m}. \]  
(39)
Here, we have the soft symbol vector $\tilde{x}_n = [\tilde{x}_{n,1}, \ldots, \tilde{x}_{n,m}, \ldots, \tilde{x}_{n,M}]^T$ by using (37), and the first iteration is completed.

In the case of the second iteration or later iterations ($k \neq 1, \ldots, k \neq 0$), $\lambda_n$ is modeled as
\[ \lambda_n = [\Theta_n - I_n] x + \phi_n, \]  
(40)
where we have
\[ I_n = \gamma \sum_{i=1, i \neq n}^{N} P_i \text{diag} \left[ \frac{\tilde{x}_{i,1}}{x_1}, \ldots, \frac{\tilde{x}_{i,m}}{x_m}, \ldots, \frac{\tilde{x}_{i,M}}{x_M} \right]. \]  
(41)
where $P_i$ denotes a matrix that consists of the off-diagonal elements of $h^n_{n,i} h^n_i$. In this case, $\Xi_n$ and $\Phi_n$, respectively, are given by
\[ \Xi_n = \frac{1}{\gamma N_0} \Theta_n^{-1} [\Theta_n - I_n], \]  
(42)
\[ \Phi_n = \frac{1}{\gamma N_0} [\Theta_n - I_n]^H \Theta_n^{-1} [\Theta_n - I_n]. \]  
(43)
As a special case where the soft symbol vector is perfect ($\tilde{x} = x_n$), we have
\[ \Xi_n = \Theta_n^{-1} (\Theta_n^D) = \frac{1}{\gamma N_0} \Theta_n^{-1} \Theta_n^D, \]  
(44)
\[ \Phi_n = (\Theta_n^D)^H \Omega_n^{-1} (\Theta_n^D) = \frac{1}{\gamma N_0} \Theta_n^D \Theta_n^{-1} \Theta_n^D, \]  
(45)
where $\Theta_n^D$ is a diagonal matrix with the diagonal elements of $\Theta_n$.

When the large system limit is satisfied, $\Theta_n$ can be regarded as a diagonal matrix owing to the strong channel hardening effect of (32). As a result, (42) and (44) approach an identity matrix $I_{M}$ while (43) and (45) approach a diagonal matrix as follows:
\[ \Xi_n \approx \frac{1}{\gamma N_0} I_M, \quad \Phi_n \approx \frac{N - 1}{N_0} I_M. \]  
(46)
Thus, we may approximately use (37) for soft symbol generation even in the case of the second iteration or later iterations.

When it is difficult to satisfy the large system limit, the tightness of the channel hardening effect of (32) is degraded by the presence of strong off-diagonal elements in $\Theta_n$. Then, the abnormal noise enhancements occur based on the correlation among $\alpha_{n,1}, \ldots, \alpha_{n,M}$ as in the case of LLR-GaBP. As a result, the soft symbol generator of (37) cannot calculate the conditional expectation of (13) accurately owing to the modeling errors in the MF belief.

3.2 NS Method for Suppression of the Outliers

The most vital issue with regard to improving the convergence property of GaBP iterative detection is how to deal with the abnormal noise enhancements in the prior belief. Here, we should re-interpret the noise enhancements as belief outliers from the ideal distribution under the large system limit. Then, the NS method can be also interpreted as the technique to suppress the occurrence probability of outliers.

Figure 2 shows the contribution of the NS method when we use a $\sqrt{N} \times \sqrt{N}$ square array of $N$ equally spaced RX antennas on the BS side. First, RX antenna indices $\{1, \ldots, N\}$ are classified into $L$ sets $\{S_l, l = 1, \ldots, L\}$ to maximize the minimum distance between two arbitrary RX antennas in each set, where $|S_l| = N/L$ is the number of elements in $S_l$. In the NS method, the beliefs corresponding to RX antenna indices are successively updated for each set $S_l$. Consequently, the minimum normalized distance between two adjacent RX antennas $d$ becomes longer, and $\rho^d$ becomes lower ($0 \leq \rho < 1$). This operation enables us to mitigate the harmful impacts of spatial fading correlation by partially updating the components of $H^H H$ along with successive soft interference cancellation.

Algorithm 1 presents the GaBP algorithm with the NS method. In the $l$-th inner iteration, $\{\alpha^{(k)}_{n,i} | i \in S_l\}$ are updated, and we assume that one outer iteration is completed when all the beliefs are updated by $L$ inner iterations. When the number of outer iterations is $K$, the number of all iterations, including inner iterations, becomes $KL$, but the number of updated beliefs in each inner iteration is $|S_l| = N/L$. Therefore, the NS method does not change the overall computational complexity, and the outer iteration number is equivalent to that of the traditional GaBP. As a special case of $L = 1$, we have the traditional GaBP in Algorithm 1.

Figure 3 shows the distribution of beliefs when $x_m = +\sqrt{E_s}$. In Fig. 3(a), a large number of outliers occur from the ideal distribution owing to the approximation errors. On the other hand, the NS method can increase the number of opportunities that the BP-based algorithm can converge without the dispersion of beliefs by partial update with the aid of successive interference cancellation, and thus suppress the occurrence of outliers in each iteration. However, persistent outliers remain in Fig. 3(b), which generate a fatal error of the soft symbol, and the convergence property of GaBP is

Fig. 2 Contribution of NS method in the $4 \times 4$ square array of equally spaced $N = 16$ RX antennas with $L = 4$. 

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NS methods can be also given. As a special case of we propose a new criterion for adaptive belief scaling corrective detection. To mitigate the harmful effects of outliers, errors, resulting in ill convergence behavior of GaBP itera-

tious, there is a possibility of including strong outliers. When the large system limit is not satisfied in practical MUD scenarios, there is a possibility of including strong outliers.

4. Adaptively Scaled Belief

When the large system limit is not satisfied in practical MUD scenarios, there is a possibility of including strong outliers in each iteration. Consequently, a fatal error of the soft symbol, i.e., a wrong hard decision symbol which is an assumed maximum error, is generated owing to the approximation errors, resulting in ill convergence behavior of GaBP iterative detection. To mitigate the harmful effects of outliers, we propose a new criterion for adaptive belief scaling corresponding to the instantaneous channel matrix $H$. From this section, the notation in Algorithm 1 is utilized based on the use of NS methods. As a special case of $L = 1$, ASB without NS methods can be also given.

Algorithm 1 GaBP with the NS method

<table>
<thead>
<tr>
<th>Initialization</th>
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<tbody>
<tr>
<td>1: $\hat{x}_{n,m}^{(0)} = 0$, $\forall n = 1, \ldots, N$, $\forall m = 1, \ldots, M$</td>
</tr>
<tr>
<td>2: $\hat{y}_{n,m}^{(0)} = 0$, $\forall n = 1, \ldots, N$, $\forall m = 1, \ldots, M$</td>
</tr>
<tr>
<td>3: $\mathcal{A}_0 = {1, \ldots, N}$, $\mathcal{B}_0 = \emptyset$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>4: for $k = 1$ to $K$ do</td>
</tr>
<tr>
<td>5: for $l = 1$ to $L$ do</td>
</tr>
<tr>
<td>6: $\mathcal{A}<em>l = \mathcal{A}</em>{l-1} \setminus \mathcal{S}_l$, $\mathcal{B}<em>l = \mathcal{B}</em>{l-1} \cup \mathcal{S}_l$</td>
</tr>
<tr>
<td>7: for all $n \in \mathcal{S}_l$ do</td>
</tr>
<tr>
<td>8: for $m = 1$ to $M$ do</td>
</tr>
</tbody>
</table>

Computation of beliefs

<table>
<thead>
<tr>
<th>9: $\hat{x}<em>{n,m}^{(k)} = h</em>{n,m}\hat{x}<em>{n,m}^{(k-1)} + h</em>{n,m}y_{n,m}^{(k)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10: $\gamma_{n,m}^{(k)} = \frac{\sum_{k=1}^{\infty} E_k}{\sum_{n} h_{n,m}^2} \left( \text{LLR} - \text{GaBP} \right)$</td>
</tr>
<tr>
<td>11: $\alpha_{n,m}^{(k)} = \gamma_{n,m}^{(k)} h_{n,m} y_{n,m}^{(k)}$</td>
</tr>
<tr>
<td>12: $\beta_{n,m}^{(k-1), l} = \sum_{i \in \mathcal{A}<em>l \setminus {n}} \alpha</em>{i,m}^{(k-1)} + \sum_{j \in \mathcal{B}<em>l \setminus {n}} \alpha</em>{j,m}^{(k)}$</td>
</tr>
</tbody>
</table>

Computation of soft symbols

<table>
<thead>
<tr>
<th>13: $\lambda_{n,m}^{(k-1), l} = \beta_{n,m}^{(k-1), l}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14: $\hat{y}<em>{n,m}^{(k-1), l} = \sqrt{\frac{E_k}{2}} \left( \text{tanh} \left[ \Re \left{ \lambda</em>{n,m}^{(k-1), l} \right} \right] + \text{j} \text{tanh} \left[ \Im \left{ \lambda_{n,m}^{(k-1), l} \right} \right] \right)$</td>
</tr>
<tr>
<td>15:</td>
</tr>
<tr>
<td>16: end for</td>
</tr>
<tr>
<td>17: end for</td>
</tr>
<tr>
<td>18: end for</td>
</tr>
<tr>
<td>19: $\hat{x}<em>{n,m}^{(k)} = \hat{x}</em>{n,m}^{(k-1), L}$</td>
</tr>
<tr>
<td>20: end for</td>
</tr>
</tbody>
</table>

Detection

| 21: for $m = 1$ to $M$ do |
| 22: $\hat{s}_m = \sqrt{\frac{E_k}{2}} \left( \text{sgn} \left[ \Re \left\{ \sum_{n=1}^{N} \alpha_{n,m}^{(k)} \right\} \right] \right.$ |
| 23: $+ \text{j} \text{sgn} \left[ \Im \left\{ \sum_{n=1}^{N} \alpha_{n,m}^{(k)} \right\} \right] \right) \rightarrow \text{Hard dec.}$ |
| 24: end for |

Terminate

Degraded. To directly mitigate the harmful effects of the persistent outliers, ASB is proposed in Sect. 4.

4.1 Criterion for Adaptive Belief Scaling According to Instantaneous Channel State

Let us focus on the behavior of the prior belief that is modeled as

$$\lambda_{n,m}^{(k-1), l} = \omega_{n,m}^{(k-1), l} \hat{x}_{m}^{(k-1), l} + \omega_{n,m}^{(k-1), l},$$

where $\omega_{n,m}^{(k-1), l}$ and $\omega_{n,m}^{(k-1), l}$ are the effective gain and noise, respectively. In the case of NS belief, according to Algorithm 1, $\omega_{n,m}^{(k-1), l}$ and $\omega_{n,m}^{(k-1), l}$ are given by

$$\omega_{n,m}^{(k-1), l} = \sum_{i \in \mathcal{A}_l \setminus \{n\}} \gamma_{i,m}^{(k-1)} |h_{i,m}|^2 + \sum_{j \in \mathcal{B}_l \setminus \{n\}} \gamma_{j,m}^{(k-1)} |h_{j,m}|^2,$$

$$\omega_{n,m}^{(k-1), l} = \sum_{i \in \mathcal{A}_l \setminus \{n\}} \gamma_{i,m}^{(k-1)} |h_{i,m}|^2 + \sum_{j \in \mathcal{B}_l \setminus \{n\}} \gamma_{j,m}^{(k-1)} |h_{j,m}|^2.$$
where we define
\[ \xi^{(k-1, l)}_{n,m} = \frac{k^{(k-1, l)}}{\omega^{(k-1, l)}_{n,m}}. \] (51)

A question arising here is how to determine the appropriate value of the parameter \( b \). If we assign a very small value to \( b \), the derived soft symbol will be small. Thus, significant interference cancellation effects cannot be realized. A typical approach for determining the scaling parameter \( b \) is to create a look-up table (LUT) according to the received SNR through computer simulations, as described in [22]. However, the scaling factor in (50) is substantially fluctuated based on \( \omega^{(k-1, l)}_{n,m} \) even though \( b \) is optimized. Thus, the SNR differs among beliefs \( \lambda^{(k-1, l)}_{n,m} \) owing to the behavior of random MIMO channels \( H \). To stabilize the fluctuations of the matrix \( H \), we define an adaptively scaled belief (ASB) that is normalized by the effective gain of (47) before the scaling as \( b = a/(\zeta_0\omega^{(k-1, l)}_{n,m}) \). The ASB is obtained by
\[ \lambda^{(k-1, l)}_{n,m} = \frac{a}{\zeta_0\omega^{(k-1, l)}_{n,m}} \lambda^{(k-1, l)}_{n,m}, \] (52)
where \( a \) denotes the scaling parameter after the stabilization. Note that the normalization factor \( \zeta_0\omega^{(k-1, l)}_{n,m} \) is computed by (48) on the BS side with the knowledge of the channel matrix \( H \), and its computational complexity is \( O(MN) \). From (14), (37), and (52), the soft symbol \( \tilde{x}_{n,m} \) generated from the ASB is given by
\[ \Re \left\{ \tilde{x}^{(k-1, l)}_{n,m} \right\} = c \tanh \left[ \frac{a}{c} \Re \left\{ x_m + \xi^{(k-1, l)}_{n,m} \right\} \right], \] (53)
\[ \Im \left\{ \tilde{x}^{(k-1, l)}_{n,m} \right\} = c \tanh \left[ \frac{a}{c} \Im \left\{ x_m + \xi^{(k-1, l)}_{n,m} \right\} \right]. \] (54)

To clarify the relationship between soft symbol \( \tilde{x}_{n,m} \) and \( a \), Fig. 4 characterizes (53) with the ASB of several scaling parameters \( a \), where \( \Re \left\{ x_m \right\} = +c \). For the ease of visualization, we focus on the real part of (53) on the premise of Gray code mapping rule. Here, the statistical behavior of \( \Re \left\{ x^{(k-1, l)}_{n,m} \right\} \) is roughly classified into two cases as follows:

- **Case 1 (C1):** \( \Re \left\{ x_m \right\} \cdot \Re \left\{ \xi^{(k-1, l)}_{n,m} \right\} \geq 0 \),
- **Case 2 (C2):** \( \Re \left\{ x_m \right\} \cdot \Re \left\{ \xi^{(k-1, l)}_{n,m} \right\} < 0 \);

C2 is sub-divided as

- **Case 2-1 (C2-1):** \( \Re \left\{ \xi^{(k-1, l)}_{n,m} \right\} \leq c \),
- **Case 2-2 (C2-2):** \( c < \Re \left\{ \xi^{(k-1, l)}_{n,m} \right\} \leq 2c \),
- **Case 2-3 (C2-3):** \( 2c < \Re \left\{ \xi^{(k-1, l)}_{n,m} \right\} \),

where the signs of the beliefs in C1 and C2-1 are identical to the sign of \( \Re \left\{ x_m \right\} \), and they improve the reliability of the iterative detection. On the other hand, the signs of the beliefs in C2-2 and C2-3 are different from the sign of \( \Re \left\{ x_m \right\} \), and they degrade the convergence property of the iterative detection. The beliefs in C2-2 exceed the decision threshold (outliers), and the beliefs in C2-3 exceeds the adjacent constellation point (strong outliers) due to abnormal noise enhancements, as shown in Fig. 4.

From the symbol generation curves in Fig. 4, the small values of \( a \) may suppress the harmful effects of outliers by adjusting the scaling parameter \( a \) with the assistance of the adaptive normalization processing.

### 4.2 Criterion for Scaling Parameter \( a \) in ASB

To design a criterion of the scaling parameter \( a \), let us focus on the occurrence probability of the beliefs classified in C2-2 and C2-3 in each outer iteration in Fig. 5. The number of MF-GaBP outer iterations is 16 and \( \rho = 0.5 \). Further, TX \( E_s/N_0 \approx -3 \) dB, which correspond to the low SNR region and high SNR region, respectively. The occurrence probabilities of C2-2 and C2-3 are compared with

- **MF-GaBP:** is defined in Sect. 3.1.
- **\( b \):** Typical scaling is defined in (50), where the scaling parameter \( b \) is determined by LUT according to the TX \( E_s/N_0 \) through preliminary computer simulations,
Fig. 5 Occurrence probability of beliefs classified in C2-2 and C2-3 in each MF-GaBP outer iteration at $E_s/N_0 = -3$ and 3 dB. The antenna configuration is $N = M = 36$, the correlation coefficient $\rho$ is 0.5, and the number of sets $L$ is 6 in the NS method. Note that the the NS method was used in all cases. Thus, the NS method is compatible with the scaling method.

- w/ ASB : is defined in (52).

The occurrence of C2-2 and C2-3 causes bit detection errors. Moreover, C2-2 and C2-3 are not independent events. The occurrence of C2-3 significantly induces the presence of C2-2.

In terms of the occurrence probability, C2-2 is dominant over C2-3. Therefore, the suppression of C2-2 effectively improves the BER performances. As can be seen in the low SNR region (a), ASB ($a = 2.0$) achieves the best performance compared to the typical scaling operation by $b$ thanks to the adaptive normalization processing for stabilizing the fluctuation of $H$. In addition, $a = 2.0$ can efficiently suppress the harmful impact of C2-2 owing to strong interference cancellation with the assistance of the wide hard-valued region. However, a floor is observed at the TX $E_s/N_0 = 3$ dB in (b) due to the strong residual outliers of C2-3 in (d), while its level is much lower than that of the typical scaling operation.

To mitigate the impact of the strong outliers of C2-3, we may assign a lower value to $a$. As the value of $a$ becomes smaller, the occurrence of C2-3 rapidly suppressed within a few iterations with the aid of the soft-valued region expansion. Although it is difficult to read the values in Fig.5, ASB with $a = 1.0$ achieves the best performance among the cases of ASB with $a = 1.0, 1.5, 2.0$ in the first and second iterations. However, the assignment of quite small $a$ ($a < 1.0$) is not effective because of ineffective interference cancellation.

In (d), with ASB ($a = 1.5$), the assignment of intermediate value to $a$ can suppress the occurrence of C2-3 rapidly, resulting in suppression of the floor in (b). The results shown in (b) and (d) indicate that the suppression of C2-3 is essential for the mitigation of the floor level of C2-2. However, as mentioned previously, ASB with $a = 1.5$ is inferior to ASB with $a = 2.0$ in the low SNR region because of an insufficient convergence due to weak interference cancellation. The observations made from (a), (b), and (d) suggest that we should assign

- lower values to $a$ in order to suppress the floor of C2-3 by reducing the probability of a wrong hard decision in the early iterations, and
- higher values to $a$ in order to suppress the floor of C2-2 by generating the correct hard decision symbol in the later iterations.

However, it is hard to mathematically optimize the dynamics of $a$ because we must focus on the instantaneous values as mentioned in Sect. 2.4. It is substantially equivalent to the rigorous analysis on the convergence property of the BP-based algorithm.

Considering the above discussion, we propose dynamic assignment of the scaling parameter $a$ in the range of 1.0 to
Algorithm 2 ASB in GaBP with NS method

Computation of ASB
1: \( \omega_{n,m}^{(k-1),l} = \sum_{i \in A_l \setminus \{n\}} \gamma_{l,m}^{(k-1)} |h_{i,m}|^2 + \sum_{j \in B_l \setminus \{n\}} \gamma_{j,m}^{(k)} |h_{j,m}|^2 \)
2: \( a = \frac{1 + \frac{k}{K}}{a} \)
3: \( \beta_{n,m}^{(k-1),l} = \frac{a}{E_{\omega_{n,m}^{(k-1),l}}} \beta_{n,m}^{(k)} \)

Computation of soft symbols
4: \( x_{n,m}^{(k-1),l} = \sqrt{E_{\omega_{n,m}^{(k-1),l}}} \left( \tanh \left( \Re \left\{ \lambda_{n,m}^{(k-1),l} \right\} \right) \right) \)
5: \( + j \tanh \left( \Im \left\{ \lambda_{n,m}^{(k-1),l} \right\} \right) \)

2.0 with an increase in the number of iterations. Thus, the simplest criterion involves linear increase, given by

\[
a = \frac{1}{a} + \frac{k}{K}.
\] (55)

The value of \( a \) is \( 1 + \frac{1}{K} \) in the first iteration and it reaches 2 in the \( K \)-th (final) iteration with an interval of \( \frac{k}{K} \). Based on ASB (linear), curves of the linear increase in \( a \) can be plotted. Owing to the suppression of C2-3 with small \( a \) in the early iterations, the floor level of C2-2 in Fig. 5(b) is significantly suppressed compared to that of ASB with \( a = 1.5 \). In the low SNR region (a), we can also confirm that ASB (linear) achieves the best performance.

When we use ASB in GaBP with NS method, the equations in lines 13–15 of Algorithm 1 in Sect. 3 are replaced with lines 1–5 of Algorithm 2.

5. Simulation Results

Computer simulations were conducted to characterize the validity of ASB in GaBP for large MU-MIMO detection under insufficient large system limit conditions. The channel state information (CSI) is perfectly known at the BS side. The average RX power from each TX antenna is assumed to be identical on the basis of slow TX power control. The modulation scheme is Gray-coded QPSK. The number of outer iterations in GaBP is 16. The number of sets \( L \) is \( \sqrt{N} \) in the NS method. Note that the NS method was used in all cases except linear minimum mean square error (MMSE) detector [3]. Thus, the NS method is compatible with the scaling method. The scaling parameter \( a \) is 1.5 in ASB when \( a \) is fixed.

5.1 BER Performance

The BER performances of LLR- and MF-GaBP for uncoded case in correlated MIMO channels (\( \rho = 0.5 \)) are shown in Fig. 6. The number of RX antennas on the BS side is \( N = 25 \) and 36, and the number of UEs (\( M \)) is assumed to be equal to \( N \). These antenna configurations cannot satisfy the large system limit, especially in correlated MIMO channels. The performances are compared with

- LLR- and MF-GaBP: are defined in Sect. 2.3 and 3.1, respectively,
- w/ Typical scaling: is defined in (50), where \( b \) is determined by LUT according to TX \( E_s/N_0 \) through preliminary computer simulations.
- w/ ASB (\( a = 1.5 \)): is defined in (52), where \( a \) is 1.5.
- w/ ASB (linear): is defined in (52), where the dynamics of \( a \) is determined by (55).
- Linear MMSE: is drawn as a baseline performance.
- EP in [9]: is drawn as a comparison with the leading-edge method, which was proposed in [9], [10]. The damping factor is 0.2 as shown in [9], \( K = 16 \).

Compared to the conventional LLR- and MF-GaBP, the typical scaling operation (w/ Typical scaling) suppresses the error floor level; however, the suppression is not sufficient.
On the other hand, ASB can significantly suppress the error floor level owing to the adaptive normalization processing according to the instantaneous channel matrix $H$. The most attractive feature is that ASB with the linear criterion (w/ ASB (linear)) can further suppress the error floor level by mitigating the harmful impact of strong residual outliers. More specifically, our proposed belief can suppress the error floor down to $\text{BER} = 10^{-5}$ in any of these cases. In addition, the conventional linear MMSE detector cannot sufficiently detect the MIMO signal due to noise enhancements, which is caused by spatial filtering. Even with the EP-based algorithm, it is difficult to suppress the approximation errors without any scaling methods. These results confirm the significance of the scaling methods in practical scenarios.

Let us shift our focus to the other MIMO antenna configurations in Fig. 7. These figures show $E_s/N_0$ required to guarantee $\text{BER} = 10^{-5}$ according to $M$ in the cases of $N = 25$ and 36. Compared to the Linear MMSE detector, the conventional LLR- and MF-GaBP cannot support more of the number of TX antennas because of lack of matrix inversion operation. On the other hand, the proposed method can drastically increase the spatial multiplexing gain in lower $E_s/N_0$ region and allow us to utilize up to about $M = 27$ ($N = 25$) and more than $M = 40$ ($N = 36$). Owing to the effective interference cancellation with the assistance of ASB, GaBP successfully perform even in overloaded MIMO channels, where the number of TX antennas exceeds the degree of freedom of RX, i.e. $M > N$.

Another point we need to note is coded case. The BER performances of LLR- and MF-GaBP for coded case in correlated MIMO channels ($\rho = 0.5$) are shown in Fig. 8. Low density parity check (LDPC) code of half rate and length 1944 bits used in the IEEE 802.11n standard is applied as the channel code. In Fig. 8(b), the LLR belief is calculated only in the last iteration of iterative detection even in MF-GaBP for yielding LLR to the channel decoder. Even when we employ channel codes, the belief outliers lead to error floors at high $E_s/N_0$ in the conventional LLR- and MF-GaBP, whereas ASB can suppress the error floor level and improve the cliff position. Note that the BER performance degradation for MF-GaBP is very small compared to that of LLLR-GaBP on the promise of ASB. This result indicates that when the large system limit is not satisfied, LLR belief no longer has any advantages over MF belief in spite of expensive computational efforts. On the other hand, the EP-based algorithm obtains about 1.5 dB gain at $\text{BER} = 10^{-5}$ as compared with the proposed method (w/ ASB (linear)) in contrast to the uncoded case in Fig. 6. This is because that
Table 1  Approximate number of real multiplication operations in a $N \times M$ MIMO channel.

<table>
<thead>
<tr>
<th>Detection method</th>
<th>Modulation</th>
<th>Approximate number of real multiplication operations $N = M = 36, K = 16$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LLR-GaBP</td>
<td>BPSK</td>
<td>$8MN + 4MN^2 + 4MN^3$</td>
</tr>
<tr>
<td></td>
<td>QPSK</td>
<td>$12MK + 8MN^2 + 4MN^3$</td>
</tr>
<tr>
<td>MF-GaBP</td>
<td>BPSK</td>
<td>$8MN + 4MN^2 + 4MN^3$</td>
</tr>
<tr>
<td></td>
<td>QPSK</td>
<td>$12MK + 8MN^2 + 4MN^3$</td>
</tr>
<tr>
<td>LLR-GaBP w/ ASB</td>
<td>BPSK</td>
<td>$8MN + 4MN^2 + 4MN^3$</td>
</tr>
<tr>
<td></td>
<td>QPSK</td>
<td>$12MK + 8MN^2 + 4MN^3$</td>
</tr>
<tr>
<td></td>
<td>QPSK</td>
<td>$12MK + 8MN^2 + 4MN^3$</td>
</tr>
</tbody>
</table>

is improved significantly because $\omega_{n,m}^{(k-1, l)}$ is calculated only in the first outer iteration ($k = 1$). Moreover, although we should consider the computational complexity to estimate the noise variance on the BS side in LLR-GaBP, the noise variance estimator is not required for MF-GaBP with ASB because the contribution of $N_0$ is disappeared by the normalization processing [24] as

$$j_{n,m}^{(k-1, l)} = a \left[ \sum_{i \in A_k \setminus \{n\}} h_{i,m}^* \tilde{y}_{i,m}^{(k-1, l)} + \sum_{j \in B_k \setminus \{m\}} h_{j,m}^* \tilde{y}_{j,m}^{(k-1, l)} \right] c \left[ \sum_{i \in A_k \setminus \{n\}} |h_{i,m}|^2 + \sum_{j \in B_k \setminus \{m\}} |h_{j,m}|^2 \right].$$

This is an advantage in the hardware implementation.

6. Conclusions

We proposed a new criterion for adaptive belief scaling according to the instantaneous channel matrix $H$ in GaBP iterative detection for large MU-MIMO. GaBP suffers from outliers induced in each iteration owing to the modeling errors in the prior belief due to the fact that the law of large number does not work well when the large system limit is not satisfied. Consequently, GaBP is subject to ill-convergence behavior of iterative detection. To mitigate the harmful impact of outliers, we introduced the concept of scaling methods. A novel adaptively scaled belief (ASB) was defined to stabilize the dynamics of random MIMO channel $H$. An advantage of ASB is that a noise variance estimator is not required for MF-GaBP. Through computer simulations, we verified the validity of the proposed ASB in terms of the significant suppression of the BER floor, even in highly correlated MIMO channels.

As one of our future studies, we are considering the application of our proposed techniques to higher-order modulation schemes because such type of modulations are widely applied to wireless communication systems.

Acknowledgements

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References


†There is a possibility that the difference between our proposed method and EP in Fig. 8 can be compensated by using an appropriate whitening filter in the final iteration although the filtering process requires matrix inversion.


