A General Perfect Cyclic Interference Alignment by Propagation Delay for Arbitrary X Channels with Two Receivers

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SUMMARY Interference alignment (IA) in temporal domain is important in the case of single-antenna vehicle communications. In this paper, perfect cyclic IA based on propagation delay is extended to the $K \times 2$ X channels with two receivers and arbitrary transmitters $K \geq 2$, which achieves the maximal multiplexing gain by obtaining the theoretical degree of freedom of $2K/(K + 1)$. We deduce the alignment and separability conditions, and propose a general scheme which is flexible in setting the index of time-slot for IA at the receiver side. Furthermore, the feasibility of the proposed scheme in the two/three-Euclidean space is analyzed and demonstrated.

key words: $K \times 2$ channel, propagation delay, cyclic interference alignment, degree of freedom, feasibility

1. Introduction

Interference alignment (IA) technique [1] has become one of the most promising approaches, which overlaps the interference in the same signal space to increase the multiplexing gain. It has been widely applied in many models such as interference channels (IC) [1] and X channels (XC) [2] and helps achieve the maximal multiplexing gain indicated by the concept of degree of freedom (DoF).

Seeking IA in the temporal domain attracted some attentions such as [3]–[8], which utilizes the PD difference among links and is suitable for the scenario of long PD communications including underwater acoustic networks (UANs). In [9], perfect PD-based cyclic IA was first studied for the $2 \times 2$ XC and $K$-user IC to achieve the DoF of $4/3$ and $K/2$, respectively. Recently, we investigate the perfect IA based on proper PD structure for $K \times 2$ XC in [10], while further node placement analysis is given in [11]. Moreover, for the three-user X channel, since perfect IA is impossible [12], we propose a feasible scheme with one extra time slot to achieve the maximum DoF by allowing one more message based on the cyclic IA approach [13].

However, [10] provided only one feasible PD pattern with perfect IA for $K \times 2$ XC and pointed out that the $K = 2$ case is a special case of the cyclic IA [9]. It is natural to ask if the perfect PD-based cyclic IA can be extended to the general $K \times 2$ XC, which leads us to this article.

In this work, we successfully extend the cyclic IA technique for XC from the basic $2 \times 2$ case to the general $K \times 2$ following the framework of the cyclic PD-based model [9]. The alignment and separability conditions of perfect cyclic IA are deduced, which achieves the DoF of $2K/(K + 1)$. Compared to [10], a general scheme with $K + 1$ PD patterns is provided, where the index of time-slot for perfect cyclic IA at the receiver side can be arbitrarily determined by a system parameter $K_1$. Furthermore, the feasibility condition is deduced in the Euclidean space.

2. System Model

The $K \times 2$ XC has $K$ ($K \geq 2$) source nodes $S_k, \forall k \in \{1, 2, \ldots, K\}$ and two destination nodes $D_j, \forall j \in \{1, 2\}$, where each node has single antenna. The message from $S_k$ to $D_j$ is denoted by $W_{kj}$. Totally, there are $M = 2K$ independent and different messages. Perfect synchronization is assumed among all communication nodes.

Equally sized time-slots are partitioned and normalized for the channel, while each message occupies one time-slot. The transmission is divided into $n \in \mathbb{N}$ consecutive time-slots as a cycle, i.e., the cyclic delays are unrolled over time. The PD between any $S_i$ and $D_j$ are assumed to be static and non-negative integer multiples of one time-slot. The channel access is stationary over a period of $n$ consecutive time-slots. Within the stationary period, delayed messages are cyclically right-shifted. After $n$ time-slots new messages are transmitted and the procedure repeats again.

Similar to [9], cyclic right-shift by polynomials in $x$ modulo $x^n - 1$ is used for modeling. Single time-slot in the period of $n$ time-slots is addressed by offsets $x^0, x^1, \ldots, x^{n-1}$, from 0 (no offset) to $n - 1$ (maximal offset). Here zero-offset indicates a reference PD. The integer PD between $S_i$ to $D_j$ is denoted by $\tau_{kj} \in \mathbb{D} = \{x^i | i = 0, 1, \ldots, n - 1\}$. We define the following $K \times 2$ PD matrix $D$ with elements $d_{kj}$ in polynomial form.
We assume unit-gain channels and zero noises as well as tent, we only discuss this DoF-optimal configuration based Otherwise, a larger period will decrease the achieved DoF, $\frac{\text{DoF}}{\text{n}}$. Let $\text{LETTER}$ time-slots at the decoder side should be equal to $n$.

From the existing result of general XC [2], the maximum achievable DoF of $K \times 2$ XC is $2K/(K+1)$. In the PD-based model, we have $M = 2K$ different messages to convey and decode over a period of $n$ time-slots. To achieve the theoretical DoF of $2K/(K+1)$, the minimal number of consecutive time-slots at the decoder side should be equal to $n = K + 1$. Otherwise, a larger period will decrease the achieved DoF, which makes the scheme suboptimal. In the following content, we only discuss this DoF-optimal configuration based on perfect cyclic IA over the above PD-based model.

Since each decoder needs $K$ time-slots to separate its required $K$ messages, only one time-slot is available for perfect cyclic IA. In detail, $\mathcal{D}_1$ should align its $K$ undesired messages $W_{k,j}$, $\forall k \in \{1, 2, \cdots, K\}$, $j \neq j$ as interference into one of the total $K + 1$ time-slots, which brings the alignment conditions:

\[
\begin{align*}
d_{k,j} &\equiv d_{i,j} \bmod X^{K+1} \\
\tau_{k,j} &\equiv \tau_{i,j} + p_{i,j} \bmod K + 1
\end{align*}
\]

for $\forall k \neq l \in \{1, 2, \cdots, K\}$ and $j \neq j \in \{1, 2\}$.

Remark: To focus on the core problem of PD-based IA, we assume unit-gain channels and zero noises as well as that in [9]. In fact, different channel gains among all source-destination pairs are also allowed in the system model. In this case, channel estimation and equalization techniques should be applied at each receiver before the above decoding procedure.

### 3. A General Perfect Cyclic IA Scheme

From the existing result of general XC [2], the maximum achievable DoF of $K \times 2$ XC is $2K/(K+1)$. In the PD-based model, we have $M = 2K$ different messages to convey and decode over a period of $n$ time-slots. To achieve the theoretical DoF of $2K/(K+1)$, the minimal number of consecutive time-slots at the decoder side should be equal to $n = K + 1$. Otherwise, a larger period will decrease the achieved DoF, which makes the scheme suboptimal. In the following content, we only discuss this DoF-optimal configuration based on perfect cyclic IA over the above PD-based model.

Since each decoder needs $K$ time-slots to separate its required $K$ messages, only one time-slot is available for perfect cyclic IA. In detail, $\mathcal{D}_1$ should align its $K$ undesired messages $W_{k,j}$, $\forall k \in \{1, 2, \cdots, K\}$, $j \neq j$ as interference into one of the total $K + 1$ time-slots, which brings the alignment conditions:

\[
\begin{align*}
d_{k,j} &\equiv d_{i,j} \bmod X^{K+1} \\
\tau_{k,j} &\equiv \tau_{i,j} + p_{i,j} \bmod K + 1
\end{align*}
\]

for $\forall k \neq l \in \{1, 2, \cdots, K\}$ and $j \neq j \in \{1, 2\}$.

Theorem 1 summarizes our main contribution.

**Theorem 1.** Perfect cyclic IA achieving the DoF of $2K/(K+1)$ for the $K \times 2$ PD-based X channels exists and a general scheme of [10] is given by the following $K \times 2$ PD matrix

\[
\mathbf{D}_{K1} = \left[\begin{array}{cc}
x_1 & x_2 \\
x_1^2 & x_2^2 \\
\vdots & \vdots \\
x_1^{K-1} & x_2^{K-1} \\
x_1^K & x_2^K
\end{array}\right]
\]

where parameters $0 \leq K_1 \leq K$ and $K_2 \geq K - K_1$ are two integers indicating the time-slot index for cyclic IA at $\mathcal{D}_1$ and $\mathcal{D}_2$, respectively, and the following transmitter polynomial

\[
v_k(x) = \left\{ \begin{array}{ll}
W_{k,1} + x^{K_1 - k_1}W_{k,2} \bmod X^{K+1}, & \forall k \leq K_1 \\
W_{k,2} + x^{K_2 + k_2 - K_1}W_{k,1} \bmod X^{K+1}, & \forall k > K_1
\end{array} \right.
\]

Proof. The proof is divided into two parts. Firstly, we show the existence of perfect cyclic IA with a DoF of $2K/(K+1)$. Then we show the general feature of the proposed scheme in contrast to that in [10].

From (5) with $n = K + 1$, at $\mathcal{D}_1$, we have
The second term in the last step shows that the alignment conditions (8) are satisfied, since all the interference messages \( W_{k,2} \) are exclusively located in time-slot \( k \) and \( K \) are aligned into the time-slot \( K + 1 \), \( \forall k \in \{1,2,\ldots,K\} \), while the third term indicates that the desired message \( W_{k,2} \) is exclusively located in time-slot \( k - K_i - 1 \), \( \forall k \in \{K_1 + 1, K_1 + 2,\ldots,K\} \). Thus, the separability conditions (9)–(10) are still satisfied.

In summary, \( 2K \) different messages are collected over \( K + 1 \) time-slots, which indicates the DoF of \( 2K/\left(K + 1\right) \) is achieved by the proposed scheme. So perfect cyclic IA exists for the PD-based \( K \times 2 \) X channels. Moreover, the above deduction also shows that \( K_1 \) and \( K_2 \) indicate the time-slot index of cyclic IA at \( D_1 \) and \( D_2 \), respectively.

Then what left is the generality proof. For any given \( K \), [10] gives one feasible PD pattern. In contrast, by adjusting \( K_1 \) from 0 to \( K \), the proposed scheme can provide \( K + 1 \) feasible PD patterns. Here we show that the one in [10] is just a special case of this work. This can be verified by setting

\[
\begin{align*}
K_1 &= N, \quad K_2 = N & \text{even: } K &\neq 2N \\
K_1 &= N, \quad K_2 = N - 1 & \text{odd: } K &\neq 2N - 1
\end{align*}
\]

(15)

where \( N \equiv \lfloor (K + 1)/2 \rfloor \) is the floored integer on \( (K + 1)/2 \). For example, if \( K = 6 \), we have \( N = \lfloor 7/2 \rfloor = 3 \) and \( K = 2N \), while if \( K = 5 \), we have \( N = \lfloor 6/2 \rfloor = 3 \) and \( K = 2N - 1 \). Next we take a closer look. With even \( K = 2N \), the above setting \( K_1 = K_2 = N \) gives the following PD matrix

\[
D_N = \begin{bmatrix}
\begin{array}{cc}
X^0 & x^N \\
X^1 & x^N \\
\vdots & \vdots \\
X^{N-1} & x^N \\
X^N & x^0 \\
\vdots & \vdots \\
X^N & x^{N-1}
\end{array}
\end{bmatrix}
\]

which results in the equivalent PD matrix expression given by Eq. (18) in [10]. Correspondingly, the transmitter polynomial (12) becomes

\[
u_k(x) = \begin{bmatrix}
W_{k,1} + x^{N-k+1}W_{k,2} & \mod X^{K+1} & & \forall k \leq N \\
W_{k,2} + x^{K-k+1}W_{k,1} & \mod X^{K+1} & & \forall k > N
\end{bmatrix}
\]

which shows the same scheduling message with Eq. (19) in [10]. With odd \( K = 2N - 1 \), the above setting \( K_1 = N, K_2 = N - 1 \) gives the following PD matrix

\[
D_N = \begin{bmatrix}
\begin{array}{cc}
X^0 & x^{N-1} \\
X^1 & x^{N-1} \\
\vdots & \vdots \\
X^{N-1} & x^{N-1} \\
X^N & x^0 \\
\vdots & \vdots \\
X^N & x^1 \\
\vdots & \vdots \\
X^N & x^{N-2}
\end{array}
\end{bmatrix}
\]
which results in the equivalent PD matrix expression given by Eq. (21) in [10]. The corresponding transmitter polynomial (12) is

\[
v_k(x) = \begin{cases} 
W_{k,1} + x^{N-k+1}W_{k,2} & \text{mod } x^{N+1}, \quad \forall k \leq N \\
W_{k,2} + x^{N-k+1}W_{k,1} & \text{mod } x^{N+1}, \quad \forall k > N
\end{cases}
\]

which shows the same scheduling message with Eq. (22) in [10]. Therefore, we have proven that the scheme in [10] just a special case of this paper. In other words, the proposed scheme is a generalization of that in [10]. This completes the proof.

More discussions are provided below for better understanding of the features of this general scheme.

The PD matrix (11) can be divided into two parts. The upper part includes the first \( K_1 \) rows, which has the same PD \( x^{K_1} \) to \( D_2 \) and increasing PD \( x^0 \) to \( x^{K_1-1} \) from \( S_1 \) to \( S_{K_1} \). The lower part includes the last \( K_2 \) rows, which has the same PD \( x^{K_1} \) to \( D_1 \) and increasing PD \( x^0 \) to \( x^{K_2-1} \) from \( S_{K_1+1} \) to \( S_K \). The proportion of the upper part is determined by \( K_1 \). By adjusting \( K_1 \) from 0 to \( K \), the PD matrix has total \( K + 1 \) patterns. Specially, if \( K_1 = 0 \) or equivalently \( K_2 = K \), the upper part will be an empty matrix without dimension since \( K_1 - 1 = -1 \). In this case, the PD matrix \( D_0 \) will only have the lower part as shown on the left of (16). On the other hand, if \( K_1 = K \) or equivalently \( K_2 = 0 \), the lower part will be an empty matrix without dimension since \( K_2 - 1 = -1 \). In this case, the PD matrix \( D_K \) will only have the upper part as shown on the right of (16). Thus we have

\[
D_0 = \begin{bmatrix} x^0 & x^0 & \cdots & x^0 \\
0 & x^1 & \cdots & 0 \\
0 & 0 & \cdots & x^{K_1-1}
\end{bmatrix}, \quad D_K = \begin{bmatrix} x^0 & x^0 & \cdots & x^0 \\
1 & x^1 & \cdots & 1 \\
0 & 0 & \cdots & 0
\end{bmatrix}
\]

for \( K_1 = 0 \) and \( K_2 = 0 \), respectively. Accordingly, the input polynomial is

\[
v_k(x) = \begin{cases} 
W_{k,1} + x^{K-k+1}W_{k,2}, & \quad \forall k \leq K_1, \quad K_1 = 0 \\
W_{k,1} + x^{K-k+1}W_{k,2}, & \quad \forall k > K_1, \quad K_2 = 0
\end{cases}
\]

And the received polynomials are

\[
r_1(x) = \frac{\sum_{k=1}^{K_1} x^kW_{k,2} + \sum_{k=1}^{K_2} x^{K-k+1}W_{k,1}}{\sum_{k=1}^{K_1} x^kW_{k,1} + \sum_{k=1}^{K_2} x^{K-k+1}W_{k,2}} \mod x^{K+1}
\]

for \( K_1 = 0 \), and

\[
r_1(x) = \frac{\sum_{k=1}^{K_1} x^{K-k+1}W_{k,2} + \sum_{k=1}^{K_2} x^kW_{k,1}}{\sum_{k=1}^{K_1} x^{K-k+1}W_{k,2} + \sum_{k=1}^{K_2} x^kW_{k,1}} \mod x^{K+1}
\]

for \( K_2 = 0 \), respectively. We can see that the above PD matrices, input and received polynomials with \( K_2 = 0 \) are equal to that with \( K_2 = 0 \) if we exchange the role of \( D_1 \) and \( D_2 \). So the above cyclic IA scheme exhibits a kind of symmetric property between \( K_1 \) and \( K_2 \), i.e., a symmetry within the range of \( K_1 \) from 0 to \( K \).

On the encoder side, we have some observations from (12). When \( k \leq K_1 \), message \( W_{k,1} \) always occupies the first time slot with index 0, while the other one \( W_{k,2} \) is allocated in the time slot \( K_1 + 1 \). For example, \( W_{1,2} \) uses time slot \( K_1 \), and \( W_{K_1,2} \) uses time slot 1. When \( k > K_1 \), message \( W_{k,2} \) always occupies the time slot 0, while the other one \( W_{k,1} \) is allocated in the time slot \( K - k + 1 \). For example, \( W_{K_1+1,1} \) uses time slot \( K - K_1 \), and \( W_{K_1,1} \) uses time slot 1.

Scheduling window is defined as the spanned time-slots from the first message to the last one. When \( k \leq K_1 \), the scheduling window at \( S_k \) is from time slot 0 to time slot \( K_1 + 1 \), among which \( S_1 \) has the longest scheduling window of \( 1 + K_1 - 1 + 1 = K_1 + 1 \) time slots. When \( k > K_1 \), the scheduling window at \( S_k \) is from time slot 0 to time slot \( K - k + 1 \), among which \( S_{K_1+1} \) has the longest scheduling window of \( 1 + K - (K_1 + 1) + 1 = K - K_1 + 1 \) time slots. So the maximal length of scheduling window denoted by \( L \) is

\[
L = \max \left( \left( K_1 + 1 \right) \left( K_1 + 1 \right), 1 + (K - k + 1) \right)
\]

\[
= \max \left( K_1, K_2 \right) + 1
\]

(18)

Tables 1 and 2 show the detailed evaluation of \( K_1, K_2 \) and \( L \) for even and odd \( K \), respectively. From Table 1 we can see that there are total \( M = N + 1 \) kinds of different \( L \) for even \( K = 2N \), among which the least length occurs at \( K = N \). From Table 2 we know that there are total \( M = N \) kinds of different \( L \) for odd \( K = 2N - 1 \), among which the least length occurs at \( K = N - 1 \) and \( K = N \). So there are \( M = [K/2] + 1 \) different values of \( L \) for a general \( K \). The minimum of \( L \) is \( N + 1 = [K/2] + 1 \) for both tables, which shows the least length of scheduling window for the proposed scheme. All other values of \( L \) from \( N + 2 \) to \( K + 1 \) have two combinations of \( K_1 \) and \( K_2 \). The maximum of \( L \) is \( K + 1 \), which occurs at \( K_1 = 0 \) or \( K_2 = K \). From the implement viewpoint, a shorter scheduling window implies less complexity, since the shift register can use a smaller length. By choosing proper value of \( K_1 \), the proposed scheme can be adaptive to variant implementation requirements.

4. Feasibility in the Euclidean Space

We analyze the feasibility of the above PD-based scheme by giving the conditions of node placement in the Euclidean space. For simplicity, constant propagation speed \( (v) \) is assumed. Denote the distance between \( D_1 \) and \( D_2 \) as \( a \). The distance between \( S_1 \) and \( D_1 \) is initialized as \( d_0 = vt_0 \), where \( t_0 \) is any positive value indicating the reference PD. The distance increase step is denoted by \( d \Delta t = vs \Delta t > 0 \), where \( s \) is a scaling factor and \( \Delta t \) is the PD gap which has been normalized in the above model.

Figure 1 is used for demonstration. Denote the circle/sphere centered at \( D_1 \) with radius \( d / 2 \) by \( O_1(d) \). The most inner circles/spheres have a radius of \( d_0 \), while the increase step of radius is \( \Delta d \). The most outer circle/sphere around \( D_1 \) and \( D_2 \) has a radius of \( d_0 + K_1 \Delta d \) and \( d_0 + K_2 \Delta d \), respectively. The positions of all source nodes are also partially
shown by the cross points of related circles/spheres. Figure 1 only shows one possible candidate choice of the source nodes in the upper part. Indeed, the intersection points of the relative circles in the lower part are also feasible for the source nodes in the two-dimensional Euclidean space. So there might be many combinations of the node placement.

In the three-dimensional Euclidean space, the intersection of the relative spheres is a circle. Because any point on the intersection circle is feasible for a source node, the number of combinations of the node placement can be large enough as long as the intersection circle has a positive radius and enough high resolution. When \( K_1 = 0 \) or \( K_2 = 0 \), there will be only one circle/sphere around \( D_1 \) or \( D_2 \), respectively, on which all source nodes are located.

The feasibility condition of the proposed scheme requires that the most outer circles/spheres centered at \( D_1 \) must have at least one intersection point with the most inner circles/spheres centered at \( D_2 \), and vice versa. Mathematically, we have: \( \forall K_1 = 0, \cdots, K \) and \( \forall K_2 = K - K_1 \)

\[
\begin{align*}
& O_1(d_0 + K_1 \Delta d) \cap O_2(d_0) \neq \emptyset \\
& O_1(d_0) \cap O_2(d_0 + K_2 \Delta d) \neq \emptyset
\end{align*}
\]

which can be further expressed with \( a \) as

\[
\begin{align*}
& a - d_0 \leq d_0 + K_1 \Delta d \leq a + d_0 \\
& a - d_0 \leq d_0 + K_2 \Delta d \leq a + d_0
\end{align*}
\]

After some computations, the above feasibility condition (20) can be simplified as

\[
\max(K_1, K_2) \Delta d \leq a \leq 2d_0 + \min(K_1, K_2) \Delta d
\]

Due to the relationship \( K_2 = K - K_1 \), we can see that a smaller \( \max(K_1, K_2) \) has a wider range of \( a \). Especially, when \( K_1 = 0 \) or \( K_2 = 0 \), with fixed \( \Delta d \) the narrowest range of \( a \) is

\[
K \Delta d \leq a \leq 2d_0
\]

which also implies that \( \Delta d \leq 2d_0/K \). On the other hand, with fixed \( \Delta d \) the widest range of \( a \) is

\[
N \Delta d \leq a \leq 2d_0 + N \Delta d
\]

for even \( K = 2N \) with \( K_1 = K_2 = N \) and

\[
N \Delta d \leq a \leq 2d_0 + (N - 1) \Delta d
\]

for odd \( K = 2N - 1 \) with \( K_1 = N \) or \( K_2 = N \). Since there is no other constraint on \( \Delta d \) in (23), theoretically it can be any positive value. With proper \( d_0 \) and \( \Delta d \), the distance between \( D_1 \) and \( D_2 \) can be determined as required. However, with odd \( K = 2N - 1 \), the feasibility condition (24) implies that \( \Delta d \leq 2d_0/K \), which limits the upper bound on \( a \) to be \( 2Nd_0 = (K + 1)d_0 \). In this sense, even \( K \) is more preferred than odd \( K \). Other choice of \( K_1 \) between 0 and \( K \) provides a trade-off between the narrowest and widest ranges of \( a \). By flexibly adjusting \( K_1, d_0 \) and \( \Delta d \) (equivalently the frame length \( \Delta r \)), the feasibility condition (21) can be widely practical.

**Examples:** For underwater acoustic applications, we can set parameters as \( \nu = 1500\text{m/s} \) and \( d_0 = 1000\text{m} \). Two examples are given below.

(i) For a \( 10 \times 2 \) XC with \( K_1 = 0 \) or \( K_1 = 10 \), the feasibility condition is \( 10d_0 \leq a \leq 2d_0 = 2000\text{m} \), which gives the narrowest range of \( a \) and implies that \( \Delta d \leq 200\text{m} \).

On the other hand, with \( K_1 = 5 \), the feasibility condition is \( 5d_0 \leq a \leq 2d_0 + 5d_0 = 2000 + 5d_0 \), which gives the widest range of \( a \) with arbitrarily value of \( \Delta d > 0 \).

(ii) For a \( 9 \times 2 \) XC with \( K_1 = 0 \) or \( K_1 = 9 \), the feasibility condition is \( 9d_0 \leq a \leq 2d_0 = 2000\text{m} \), which gives the narrowest range of \( a \) and implies that \( \Delta d \leq 2000/9 \text{m} \).

On the other hand, with \( K_1 = 5 \) or \( K_2 = 5 \), the feasibility condition is \( 5d_0 \leq a \leq 2d_0 + 4d_0 = 2000 + 4d_0 \), which gives the widest range of \( a \) with \( \Delta d \leq 2d_0 = 2000\text{m} \).

Moreover, when a suboptimal \( n \) (i.e. \( n > K + 1 \)) is considered, we can expect that the feasibility can be greatly improved, since there will be more feasible patterns supported.
and the alignment conditions can be relaxed to some extent. For example, with \( n = K + 2 \), now we have two time-slots for cyclic IA of the \( K \) undesired messages, which has many combinations of possible interference grouping for cyclic IA at each destination node, in contrast with the only one combination with \( n = K + 1 \). This provides a trade-off method between the multiplexing gain and feasibility in the Euclidean space. When no perfect cyclic IA exists such as in the 3-user X-networks shown in [12], this approach can be expected to make a feasible suboptimal scheme. In fact, when more time slots are involved, more messages besides the original ones are also possible, which compensates the DoF loss to some extent. For example, with one extra time slot, by allowing one more message the maximum DoF of \( 10/6 = 5/3 \) instead of \( 9/6 = 3/2 \) is achieved based on cyclic IA for the three-user X channel in [13].

5. Conclusion

We extended the approach of perfect cyclic IA by PD to \( K \times 2 \) X channels and proposed a general scheme, which is optimal to achieve the DoF of \( 2K/(K + 1) \). The position of IA can be arbitrarily determined by the parameter \( K_1 \) from time-slot 0 to \( K \) along with the \( K + 1 \) patterns. Feasibility analysis shows that it is widely practical in the Euclidean space. This work generalizes the scope of cyclic IA for X channels in temporal domain.

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References


