SUMMARY This paper proposes low complexity resource allocation in frequency domain non-orthogonal multiple access where many devices access with a base station. The number of the devices is assumed to be more than that of the resource for network capacity enhancement, which is demanded in massive machine type communications (mMTC). This paper proposes two types of resource allocation techniques, all of which are based on the MIN-MAX approach. One of them seeks for nicer resource allocation with only channel gains. The other technique applies the message passing algorithm (MPA) for better resource allocation. The proposed resource allocation techniques are evaluated by computer simulation in frequency domain non-orthogonal multiple access. The proposed technique with the MPA achieves the best bit error rate (BER) performance in the proposed techniques. However, the computational complexity of the proposed techniques with channel gains is much smaller than that of the proposed technique with the MPA, whereas the BER performance of the proposed techniques with channel gains is only about 0.1 dB inferior to that with the MPA in the multiple access with the overloading ratio of 1.5 at the BER of $10^{-4}$. They attain the gain of about 10 dB at the BER of $10^{-4}$ in the multiple access with the overloading ratio of 2.0. Their complexity is $10^{-16}$ as small as the conventional technique.

**key words:** Non-Orthogonal Multiple Access, Frequency Domain, Adaptive subcarrier allocation, low complexity, MIN-MAX approach.

1. Introduction

Machine-type communications (MTC) have been identified as a part of the fifth generation mobile communication system and the beyond 5th generation system for the society with Internet of things (IoT). The society with the IoT needs a lot of sensor devices with wireless communication functionality, which are going to be scattered around us. Massive connectivity is demanded for the connection with those devices when the number of those devices grows extremely high. In a word, network capacity has to be increased for such massive MTC (mMTC), though amount of data sent by a device might not be huge. Many techniques have been proposed for enhancement of the wireless network capacity. For instance, multi-user multiple input multiple output (MU-MIMO) [1] and orthogonal frequency division multiple access (OFDMA) [2], that are classified into orthogonal multiple access, have been investigated. Non-orthogonal multiple access also has been considered, because non-orthogonal multiple access potentially achieves higher capacity than orthogonal multiple access. Non-orthogonal multiple access (NOMA) [3]–[9], low-density signature (LDS) [10], [11], and sparse code multiple access (SCMA) [12], [13] have been proposed as non-orthogonal multiple access techniques. Adaptive resource allocation has been proposed for non-orthogonal multiple access based on low density signature. Whereas fixed resource allocation to reduce the peak average power ratio (PAPR) has been proposed [14], most of them have investigated resource allocation and power allocation to maximize sum rate [15]–[19]. For the maximization, they try to find the solution of the constraint optimization problems.

A simpler resource allocation technique has been proposed to improve the BER performance [20]. Since subcarriers are assigned based on its gains regardless of the detector performance, this can be easily implemented. Although another adaptive resource allocation has been proposed to improve the BER performance [21], the allocation technique executes the message passing algorithm (MPA) [22], [23] for all the resource allocation patterns, and finds the best resource allocation pattern that minimizes the BER performance. However, this imposes prohibitive high computational complexity on the system.

In this paper, we propose low complexity resource allocation in frequency domain non-orthogonal multiple access where many devices access with a base station. Our proposed techniques apply channel swapping for the resource allocation, which enables our proposed techniques to be implemented with lower complexity. In addition, the proposed techniques seek better resource allocation to minimize the BER performance. The best subcarrier allocation is searched using the channel swapping based on the MIN-MAX approach in the proposed techniques, which has not been considered before, while the MIN-MAX approach has been applied even for resource allocation problems. These ways make the proposed techniques completely different from the conventional techniques. Our proposed techniques can be classified into two types. One of them seeks for nicer resource allocation with only channel gains. The other applies the message passing algorithm (MPA) only for the nice candidates of resource allocation, while the conventional technique executes the MPA for all the resource allocation patterns. The performance of those proposed techniques are evaluated by computer simulations. The proposed techniques achieve as near BER performance as the conventional technique. However, the proposed techniques can be imple-

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mented with much less computational complexity than the conventional technique.

Throughout the paper, j, and $c^*$ represent the imaginary unit, and complex conjugate of a complex number $c$. Superscript $\mathbf{T}$ and $\mathbf{H}$ indicate transpose and Hermitian transpose of a matrix or a vector, respectively. In addition, $\text{diag} \{ \mathbf{V}, \mathbf{A}_l, \mathbf{A}_{k,l} \}$ and $\mathbf{A}_{k,l}$ indicate a diagonal matrix with a vector $\mathbf{V}$ in the diagonal position, an $l$th column vector, a $k$th row vector, and a $(k, l)$ element of a matrix $\mathbf{A}$, respectively.

2. System Model

We assume that a user owns $L$ IoT devices for collecting information measured with the sensors on the devices, and one resource block is allocated to the user. In other words, the user communicates with those IoT devices through wireless communication within the allocated resource block. The number of the devices is possibly increasing regardless of the resource. In a word, the number of the IoT devices, $L$, exceeds the resource when the mMTC comes reality. The transmitters on the IoT devices are assumed to have always some information to send, which forces all the IoT devices to send their packet simultaneously for the base station. Only one antenna is installed on every IoT device and the base station, a received signal vector $\mathbf{Y} \in \mathbb{C}^{N_f}$ in the time domain can be written as,

$$\mathbf{Y} = \sum_{l=1}^{L} \mathbf{H}_l \mathbf{S}^{(n)}(l) + \mathbf{N}. \quad (5)$$

In (5), $\mathbf{N} \in \mathbb{C}^{N_f}$ represents an additive white Gaussian noise (AWGN) vector. As is shown in (5), the received signal is superposition of the transmission signals from the $L$ devices.

Throughout the paper, $j$, and $\gamma_l$ represent the imaginary unit, and complex conjugate of a complex number $\gamma_l$. Superscript $\mathbf{T}$ and $\mathbf{H}$ indicate transpose and Hermitian transpose of a matrix or a vector, respectively. In addition, $\text{diag} \{ \mathbf{V}, \mathbf{A}_l, \mathbf{A}_{k,l} \}$ and $\mathbf{A}_{k,l}$ indicate a diagonal matrix with a vector $\mathbf{V}$ in the diagonal position, an $l$th column vector, a $k$th row vector, and a $(k, l)$ element of a matrix $\mathbf{A}$, respectively.

$$\mathbf{Y} = \sum_{l=1}^{L} \mathbf{H}_l \mathbf{S}^{(n)}(l) + \mathbf{N}.$$  

(6)

In the above equation, $\mathbf{Y} \in \mathbb{C}^{N_s}$ and $\mathbf{X}$ denote a received signal vector in the frequency domain and a transmission signal vector containing all the devices' transmission signals defined as $\mathbf{X} = (x(1), \cdots, x(L))^T$. In addition, $\mathbf{S}^{(n)} \in \mathbb{C}^{N_s}$ represents an equivalent channel matrix defined as,

$$\mathbf{S}^{(n)} = \mathbf{F} \mathbf{X}^{(n)}(l). \quad (1)$$

In (1), $\mathbf{X}^{(n)}(l) \in \mathbb{C}^{N_s}$ represents a modulation signal vector sent from the $l$th device. Let $\mathbf{F}_{p,q} \in \mathbb{C}$ represent a $(p, q)$ element of the PDF matrix, the PDF matrix $\mathbf{F}$ is defined as,

$$\hat{\mathbf{F}}_{p,q} = \frac{1}{\sqrt{N_f}} e^{-j \frac{2\pi b(p-1)(q-1)}{N_f}}. \quad (2)$$

$\hat{\mathbf{F}}_{p,q}$ denotes the $p$th subcarrier for the device to send the signals, which is defined as follows.

$$\mathbf{X}^{(n)}(l) = \mathbf{C}^{(n)}_l x(l) \quad (3)$$

In (3), $\mathbf{C}^{(n)}_l \in \mathbb{R}^{N_s}$ represents a subcarrier allocation vector for the $l$th device, which is defined as $\mathbf{C}^{(n)}_l = [c^{(n)}_{l,0}, \cdots, c^{(n)}_{l,N_s-1}]^T$, where $c^{(n)}_{l,i-1} \in \mathbb{R}$ indicates the $i$th element of the vector $\mathbf{C}^{(n)}_l$. The element defines availability of the subcarrier for the device to send the signals, which is defined as follows.

$$c^{(n)}_{l,i-1} = \begin{cases} 1 & (i \text{ th subcarrier is available}) \\ 0 & (i \text{ th subcarrier is not available}) \end{cases} \quad (4)$$

As is shown in (3) and (4), the device actually only transmits the same packet in the $M$ subcarriers where $M$ indicates the number of the subcarriers allocated to an IoT device. Let $\mathbf{C}^{(n)}_l \in \mathbb{C}^{N_s \times L}$ denote a subcarrier allocation matrix, the matrix is defined as $\mathbf{C}^{(n)}_l = \left[ c^{(n)}_{l,1} \cdots c^{(n)}_{l,L} \right]$. All the devices simultaneously send their packets for the base station after the cyclic prefixes are added to those signals. The base station receives those transmission signals that have passed through multipath fading channels, where the path length is less than the cyclic prefix length. Let $\mathbf{H}_l \in \mathbb{C}^{N_f \times N_f}$ denote a channel matrix between the $l$th device and the base station, a received signal vector $\mathbf{Y} \in \mathbb{C}^{N_f}$ in the time domain can be written as,

$$\mathbf{Y} = \sum_{l=1}^{L} \mathbf{H}_l \mathbf{S}^{(n)}(l) + \mathbf{N}.$$  

(5)

In (5), $\mathbf{N} \in \mathbb{C}^{N_f}$ represents an additive white Gaussian noise (AWGN) vector. As is shown in (5), the received signal is superposition of the transmission signals from the $L$ devices. The received signal vector in the time domain is transformed into the frequency domain as,

$$\mathbf{Y} = \mathbf{F} \mathbf{Y} = \sum_{l=1}^{L} \mathbf{F} \mathbf{H}_l \mathbf{S}^{(n)}(l) + \mathbf{F} \mathbf{N}.$$  

(6)

In the above equation, $\mathbf{Y} \in \mathbb{C}^{N_s}$ and $\mathbf{X}$ denote a received signal vector in the frequency domain and a transmission signal vector containing all the devices' transmission signals defined as $\mathbf{X} = (x(1), \cdots, x(L))^T$. In addition, $\mathbf{S}^{(n)} \in \mathbb{C}^{N_s \times L}$ represents an equivalent channel matrix defined as,

$$\mathbf{F}^{(n)} = \left[ \mathbf{\Gamma} (1) \mathbf{C}^{(n)}_1 \cdots \mathbf{\Gamma} (L) \mathbf{C}^{(n)}_L \right]. \quad (7)$$

In (7), $\mathbf{\Gamma} (l) \in \mathbb{C}^{N_s \times N_s}$ denotes a diagonal matrix with the frequency responses between the $l$th device and the base station in the diagonal positions, which is expressed as follows.

$$\mathbf{\Gamma} (l) = \text{diag} \left\{ \gamma_l(0), \gamma_l(1) \cdots \gamma_l(N_s-1) \right\} \quad (8)$$

In (8), $\gamma_l(m)$ represents a frequency response in the $m$th subcarrier between the $l$th device and the base station, which is defined as,

$$\gamma_l(m) = \sum_{p=0}^{L_p-1} h_l(p) \exp \left( -j 2 \pi \frac{mp}{N_f} \right), \quad \text{where } L_p \in \mathbb{N} \text{ and } h_l(p) \in \mathbb{C} \text{ denote the number of the paths in the multipath fading channel and a complex path gain of the } p \text{th path in the channel between the } l \text{th IoT device and the base station.}$$

Even when superposition of the transmission signals sent from the $L$ devices is received at black the base station,
non-linear detectors can detect those signals. Especially, non-linear detectors based on the MPA is more useful if the equivalent channel matrix is sparse, because the detectors become less complex, while the detectors achieve almost the same performance to the maximum likelihood estimation.

For enhancing transmission performance, the adaptive resource allocation has been proposed [21]. Better subcarrier allocation is estimated by the adaptive resource allocation with the estimated channel matrix at the base station, and is informed every IoT device for the signal transmission in the next time slot. However, it needs the prohibitive high complexity that grows exponentially with the number of the devices \( L \); the MPA is executed \( Q^L \left( \frac{L}{1} \right) \) times, where \( Q \) represents cardinality of the modulation, because the exhaustive search is applied.

We propose low complexity resource allocation algorithm in the following section.

3. Low Complexity Resource Allocation

Even when a detector based on the MPA is applied to the receiver on the base station, the performance is dependent on the channel matrix in the system. This means that the detection performance could be estimated through analysis of the channel matrix in the system with the frequency domain non-orthogonal multiple access. This section proposes low complexity resource allocation based on analysis of the channel matrix. As is described above, while the MIX-MAN approach is applied to the proposed low complexity resource allocation, we propose four techniques based on different criteria, which are described in the following subsections.

3.1 Least Channel Gain Maximization With Channel Vector Swapping

This section proposes a resource allocation technique that maximizes the smallest channel gain in the equivalent channel matrix. For searching the best resource allocation, we apply vector swapping to the subcarrier allocation matrix, with which resource allocation can be implemented. The vector swapping is iterated until better resource allocation is found in the proposed technique. Let \( \tilde{\mathbf{g}}^{(n)} \) denote an equivalent channel matrix at the \( n \)th iteration stage, first of all, the position of the minimum channel gain \( \tilde{\mathbf{g}}^{(n)}_{k_n, l_n} \in \mathbb{C} \) in the equivalent channel matrix is searched as,

\[
(k_n, l_n) = \arg\min_{(k, l) \in B^{(n)}} \left| \tilde{\mathbf{g}}^{(n)}_{k, l} \right|
\]

In (10), \( k_n \in \mathbb{N} \) and \( l_n \in \mathbb{N} \) denote row and column indexes of the position of the smallest channel gain at the \( n \)th iteration stage. In addition, \( B^{(n)} \) in (10) represents a set containing the positions of the non-zero elements in the matrix \( \tilde{\mathbf{g}}^{(n)} \).

\[\text{which is defined as } B^{(n)} = \{(k, l) | \tilde{\mathbf{g}}^{(n)}_{k, l} \neq 0 \}.\]

For the vector swapping, a permutation matrix \( \mathbf{J}(l_1 \leftrightarrow l_2) \in \mathbb{N}^{L \times L} \) is introduced to swap \( l_1 \)th column and \( l_2 \)th column as follows.

\[
\mathbf{J}(l_1 \leftrightarrow l_2) = \begin{pmatrix}
1 & & \\
& \ddots & \\
& & 0 \ldots 1 \\
& & \ddots & \\
& & & 1 \ldots 0 \\
& & & & \ddots \\
& & & & & 1
\end{pmatrix}
\]

A candidate subcarrier allocation matrix \( \mathbf{C}^{(r_n)} \in \mathbb{C}^{N_s \times L} \) and a candidate equivalent channel matrix \( \tilde{\mathbf{g}}^{(r_n)} \in \mathbb{C}^{N_s \times L} \) are obtained with the permutation matrix as follows.

\[
\mathbf{C}^{(r_n)} = \mathbf{C}^{(n)} \mathbf{J}(l_n \leftrightarrow r_n)
\]

\[
\tilde{\mathbf{g}}^{(r_n)} = \mathbf{J}(1)\mathbf{C}_1^{(r_n)} \ldots \mathbf{J}(L)\mathbf{C}_L^{(r_n)}
\]

In (12) and (13), \( r_n \in \mathbb{N} \) represents a column index different from \( l_n \). Let \( B_i^{(r_n)} \) denote a set containing the positions of the nonzero elements in the \( i \)th column vector of the matrix \( \tilde{\mathbf{g}}^{(r_n)} \), the set is defined as \( B_i^{(r_n)} = \{ k | \tilde{\mathbf{g}}_{k, i}^{(r_n)} \neq 0 \} \). The position of the minimum non-zero element in the \( l_n \)th and \( r_n \)th column vectors is searched as,

\[
(k[r_n], l[r_n]) = \arg\min_{(k, l) \in B_i^{(r_n)}} \left| \tilde{\mathbf{g}}_{k, l}^{(r_n)} \right|
\]

(14)

Since the \( l_n \)th column can be swapped with all the columns except for \( l_n \)th column, we find the best column that provides the biggest value \( \tilde{\mathbf{g}}_{k[r_n], l[r_n]}^{(r_n)} \), which is written as follows.

\[
\tilde{r}_n = \arg\max_{r_n} \left| \tilde{\mathbf{g}}_{k[r_n], l[r_n]}^{(r_n)} \right|
\]

(15)

\( \tilde{r}_n \in \mathbb{N} \) represents a column index that makes the permutation provide the biggest value \( \tilde{\mathbf{g}}_{k[r_n], l[r_n]}^{(r_n)} \) in the indexes \( r_n = 1 \ldots L \). The best candidate subcarrier allocation matrix \( \mathbf{C}^{(n+1/n)} \in \mathbb{C}^{N_s \times L} \) can be obtained with the column index \( \tilde{r}_n \) as follows.

\[
\mathbf{C}^{(n+1/n)} = \mathbf{C}^{(n)} \mathbf{J}(l_n \leftrightarrow \tilde{r}_n)
\]

(16)

The above signal processing is depicted by Fig. 1 where the equivalent channel matrices with \( N_s = 4, M = 2, \) and \( L = 6 \) are shown. \( y_i(k) \) is equal to a \( (k, l) \) element of the channel matrix \( \tilde{\mathbf{g}}^{(n)} \), i.e., \( y_i(k) = \tilde{\mathbf{g}}_{k, l}^{(n)} \). The channel gain \( y_4(2) \) is assumed to be the smallest in the upper matrix in the figure. The subcarriers allocated to the 4th device.
are attempted to be reallocated to the other device \(^\dagger\). If the subcarriers are swapped with those to the 3rd device, for example, the lower equivalent channel matrix will be obtained. The smallest gain in the subcarriers for the 3rd and 4th devices is searched, which is implemented in (14). The above attempt is repeated with all the possible columns, and the smallest gains given by all the attempts are compared. When the attempt with the \(r_n\) th device attains the largest gain, which is done in (15), the subcarriers allocated to the 4th device are swapped with those of the \(r_n\) th device. The permuation matrix corresponding to the swap is selected for the best candidate subcarrier allocation matrix as shown in (16).

The subcarrier allocation matrix \(C^{(n)}\) is updated as,

\[
C^{(n+1)} = \begin{cases} 
C^{(n+1/n)} & |\tilde{r}_n^{(n)}| \leq |\tilde{r}_n^{(n+1/n)}| \\
C^{(n)} & \text{otherwise}
\end{cases}
\]

(17)

This means that the subcarrier allocation matrix is updated only if the update of the subcarrier allocation matrix makes the absolute value \(|\tilde{r}_n^{(n+1/n)}|\) bigger than that of the minimum non-zero element \(|\tilde{r}_n^{(n)}|\). Otherwise, the subcarrier allocation is not update, which means the end of the iteration in the proposed technique. Even in the following proposed techniques, if the subcarrier allocation matrix is not updated, the iteration is terminated.

The subcarrier allocation technique described in the section is called “Least Channel Gain Maximization With Channel Vector Swapping”, which is abbreviated as “CGVS”.

3.2 Least Channel Norm Maximization With Channel Vector Swapping

Since the overall BER performance is dominated by the BER performance of the worst IoT device, the improvement in the BER performance of the worst device leads the overall transmission performance improvement. Even when the MPA is applied to the receivers, transmission performance of a device greatly depends on the received signal amplitude associated with the device. In the frequency domain non-orthogonal multiple access, the norm of the equivalent channel vector for the \(l\)th IoT device can be regarded as its received signal amplitude. This section proposes a resource allocation technique that maximizes the smallest norm of the equivalent channel vector in the equivalent channel matrix.

The worst devices can be found in search for the minimum norm as follows.

\[
l_n = \arg \min_{l} \left[ \left| \tilde{r}_l^{(l)} \right| \right]
\]

(18)

In (18), \(l_n\) represents an index of the worst IoT device at the \(n\)th iteration stage. Similar to the technique proposed in the previous section, a candidate subcarrier allocation matrix and a candidate equivalent matrix are generated through the permutation in (12) and (13), respectively. Since the \(l_n\) column and the \(r_n\) column of the candidate subcarrier allocation matrix are exchanged, only the \(l_n\) column and the \(r_n\) column of the candidate equivalent matrix are affected by the permutation. The column with smaller norm is searched among the two columns as,

\[
l^{[r_n]} = \arg \min_{l_n, r_n} \left[ \left| \tilde{r}_l^{[r_n]} \right|, \left| \tilde{r}_r^{[r_n]} \right| \right]
\]

(19)

\(l^{[r_n]} \in \mathbb{N}\) is either \(l_n\) or \(r_n\). Since \(r_n\) can range from 1 to \(L\) except for \(l_n\), the best column number \(\overline{r}_n \in \mathbb{N}\) is searched as,

\[
\overline{r}_n = \arg \max_{r_n} \left| \tilde{r}_l^{[r_n]} \right|
\]

(20)

\[
C^{(n+1/n)} = C^{(n)} \cdot J \left( l_n \leftrightarrow \overline{r}_n \right)
\]

(21)

As is shown above, the minimum norm can be maximized by swapping \(l_n\)th column with \(\overline{r}_n\) column. The subcarrier allocation matrix is updated only if the best permutation increases the smallest norm of the column vector as follows.

\[
C^{(n+1)} = \begin{cases} 
C^{(n+1/n)} & |\tilde{r}_n^{(n)}| \leq |\tilde{r}_n^{(n+1/n)}| \\
C^{(n)} & \text{otherwise}
\end{cases}
\]

(22)

The subcarrier allocation technique described in the section is called “Least Channel Norm Maximization With Channel Vector Swapping (CNVS)”.

3.3 Least Channel Gain Maximization With Channel Gain Swapping

In the above sections, two columns in the subcarrier allocation matrix are swapped to search better subcarrier allocation. This section proposes to swap two terms in the subcarrier allocation matrix with keeping the number of non-zero terms in the column and the row of the equivalent channel

\[
\begin{pmatrix}
\gamma_1 (1) & \gamma_2 (1) & \gamma_3 (1) \\
\gamma_1 (2) & 0 & \gamma_4 (2) \\
0 & \gamma_2 (3) & \gamma_4 (3) \\
0 & 0 & \gamma_3 (4)
\end{pmatrix}
\begin{pmatrix}
\gamma_5 (2) & 0 & \gamma_6 (3) \\
0 & \gamma_5 (4) & \gamma_6 (4)
\end{pmatrix}
\]

Fig. 1 Vector swapping in CGVS
matrix or the subcarrier allocation matrix constant \(^\dagger\). Same to the CGVS, the proposed technique is to maximize the smallest channel gain in the equivalent channel matrix. First to the CGVS, the proposed technique is to maximize the smallest channel gain in the equivalent channel matrix or the subcarrier allocation matrix, which degrades the decoding performance of the MPA. This is the reason why those numbers are kept constant in the modification of the channel allocation matrix.

\[
\begin{pmatrix}
γ_1(1) & γ_2(1) & γ_3(1) & 0 & 0 & 0 \\
0 & γ_2(2) & γ_3(2) & 0 & 0 & 0 \\
0 & 0 & γ_2(3) & γ_3(4) & 0 & 0 \\
0 & γ_2(1) & 0 & γ_3(2) & 0 & 0 \\
0 & 0 & γ_2(3) & 0 & γ_3(4) & 0 \\
0 & γ_2(1) & 0 & 0 & γ_3(4) & 0 \\
\end{pmatrix}
\]

Fig. 2 Gain swapping in CGGS

\[C \in \mathbb{C}^{N \times L}\] can be obtained as,

\[
\Gamma(n, [k_i, l_i]) = \left[ \Gamma(1)C_n^{[k_i, l_i]} \cdots \Gamma(L)C_L^{[k_i, l_i]} \right].
\]

(28)

For the row index \(k_i\), a column index \(l_i\) \(\in \mathbb{N}\) is searched within the set \(L_{n, l_i^{(k_i)}}\) to search better element as,

\[
l_{n+1, l_i^{(k_i)}} = \arg \max_{l_i \in L_{n, l_i^{(k_i)}}} \left| \bar{\Gamma}(n, [k_i, l_i]) \right|.
\]

(29)

\(l_{n+1, l_i^{(k_i)}} \in \mathbb{N}\) denotes an index which element is the biggest in the \(k_i\)th row. Since there are some elements in the set \(K_n\), the best row index \(\bar{k}_{n+1}\) \(\in \mathbb{N}\) that maximizes the smallest equivalent channel gain can be found,

\[
\bar{\gamma}_{n+1}^{(k_i)} = \min_{k_i \in K_n} \left| \bar{\Gamma}(n, [k_i, l_i]) \right|.
\]

(30)

The best candidate subcarrier allocation matrix \(C^{(n+1) l_i^{(k_i)}} \in \mathbb{N}^{L \times L}\) is defined with the optimum row number \(\bar{k}_{n+1}\) as follows.

\[
C^{(n+1) l_i^{(k_i)}} = \begin{cases} 
0 & (k, l) = (k_n, l_n) \\
1 & (k, l) = (\bar{k}_{n+1}, 1) \\
1 & (k, l) = (k_n, \bar{l}_{n+1}) \\
C^{(n) l_i^{(k_i)}} & \text{otherwise}
\end{cases}
\]

(32)

\(l_{n+1, l_i^{(k_i)}} \in L_{n, l_i^{(k_i)}}\) denotes an index which element is the biggest in the \(k_i\)th row. Since there are some elements in the set \(K_n\), the proposed technique selects the best column for the reallocation in (31) that maximizes the minimum gain in the equivalent channel matrix.

Finally, the subcarrier allocation matrix is updated only
if the update increases the minimum term of the equivalent channel matrix.

\[
C^{(n+1)} = \begin{cases} 
C^{(n+1)/n} & \hat{\Gamma}_{k, n-1}^2 < \sigma^2_{n+1/n} \\
C^{(n)} & \text{otherwise}
\end{cases}
\]  
(33)

Hence, the equivalent channel matrix is also updated with the updated subcarrier allocation matrix \( C^{(n+1)} \).

\[
\hat{\Gamma}^{(n+1)} = \left[ \Gamma(1) C_{1}^{(n+1)} \cdots \Gamma(L) C_{L}^{(n+1)} \right]
\]  
(34)

The technique proposed in the section is called “Least Channel Gain Maximization With Channel Gain Swapping (CGGS)”

### 3.4 Least LLR Maximization With Channel Vector Swapping

In the proposed techniques described above, the subcarrier allocation is searched for maximizing the least channel gain or the least channel vector norm in the equivalent channel matrix. On the other hand, the MPA output signals can be used as better metric to select the best swapping than the channel gains, because the MPA is employed at the receiver. Hence, we apply the MPA to the subcarrier allocation technique. Because the vector swapping achieves better performance than the gain swapping as is shown afterwards, the vector swapping is applied to the resource allocation based on the MPA. As is done in the vector swapping, firstly, the signal processing described in (12) and (13) is carried out to get the candidate subcarrier allocation matrix. With the matrix, the log-likelihood ratio is calculated in the following equation.

Let \( \hat{y}_{[r_n]}^{(\beta)}(k) \) be a \( k \)th candidate of the transmission signal vector, a received signal at the \( k \)th subcarrier \( \hat{y}_{[r_n]}^{(\beta)}(k) \) in the non-orthogonal noise-free channel with the candidate subcarrier allocation matrix \( C^{(\beta)} \) is written as.

\[
\hat{y}_{[r_n]}^{(\beta)}(k) = \sum_{l=1}^{L} \hat{\Gamma}_{k, l} \hat{x}_{(l)}^{(\beta)}(l) = \hat{\Gamma}_{k}^{[r_n]} \hat{x}^{(\beta)}.
\]  
(35)

In (35), \( x_{(l)}^{(\beta)}(l) \) in \( C \) represents an \( l \)th element of the vector \( x_{(l)}^{(\beta)} \), i.e., \( x_{(l)}^{(\beta)} = [x_{(l)}(1) \cdots x_{(l)}(L)]^T \). The symbol LLR of the signal \( x_{(l)}^{(\beta)} \) can be obtained as follows.

\[
\Delta x_{[r_n]}^{(\beta), (l)}(x(l) = \alpha) = \log \frac{P(x(l) = \alpha | \hat{y}_{[r_n]}^{(\beta)}(k))}{P(x(l) = \alpha | \hat{y}_{[r_n]}^{(\beta)}(k))}
\approx \max_{x(l) = \alpha} \left\{ -\frac{1}{2} \hat{\Gamma}_{k, l}^{[r_n]} \hat{x}^{(\beta)}_{(l)} \sum_{i \in B_{[k]}^{(\beta)}} P(x(i)) \right\}
\]  
(36)

\[
-\max_{x(l) = \alpha} \left\{ -\frac{1}{2} \hat{\Gamma}_{k, l}^{[r_n]} \hat{x}^{(\beta)}_{(l)} \sum_{i \in B_{[k]}^{(\beta)}} P(x(i)) \right\}
+ \log \frac{P(x(l) = \alpha)}{P(x(l) = \alpha)}
\]  
(37)

In (36), \( \Delta x_{[r_n]}^{(\beta), (l)}(x(l) = \alpha) \in \mathbb{R}, P(a), P(Q | R), \hat{x} \in C^L \) and \( \sigma \) represent a modulation signal, a symbol LLR of the modulation signal \( x(l) = \alpha \), probability that an event \( a \) happens, conditional probability of an event \( Q \) when an event \( R \) occurred, a tentative transmission signal vector, and a reference modulation signal, respectively. In addition, \( B_{[k]}^{(\beta)} \) denotes a set containing positions of non-zero terms in the row vector \( \hat{\Gamma}_{[k]}^{(\beta)} \), i.e., \( B_{[k]}^{(\beta)} = \{ l | \hat{\Gamma}_{k, l}^{[r_n]} \neq 0 \} \). Let \( \alpha_{\max} \) denote a modulation signal that maximizes the LLR among all the modulation signal candidates except for the transmission signal \( x(\beta) \), the reliability of the \( l \)th modulation signal \( \Delta x_{[r_n]}^{(\beta)}(x(l), \alpha_{\max}) \) can be defined as [21],

\[
\Delta x_{[r_n]}^{(\beta)}(x(l), \alpha_{\max}) = \Delta x_{[r_n]}^{(\beta), (l)}(x(l) = \alpha_{\max}) + \Delta x_{[r_n]}^{(\beta), (l)}(x(l) = x(\beta))
\]  
(38)

\[
\Delta x_{[r_n]}^{(\beta), (l)}(x(l)) \in \mathbb{R}
\]  
(39)

can be regarded as reliability of the \( l \)th modulation signal in the \( k \)th subcarrier. Assuming that all the transmission signal vectors are generated with equal probability, the average reliability over all the subcarriers can be denoted as the reliability of the \( l \)th IoT device.

Since the BER performance is dominated by the BER performance of the worst device as described above, this section proposes a resource allocation technique that maximizes the minimum reliability of the transmission symbols. The minimum reliability \( \gamma_{[r_n]}^{[r_n]} \in \mathbb{R} \) can be defined as,

\[
\gamma_{[r_n]}^{[r_n]} = \min_{l} \left\{ \sum_{k=0}^{N_c-1} \sum_{k=0}^{L} \Delta x_{[r_n]}^{(\beta), (l)}(x(l)) \right\}
\]  
(40)

Since the IoT device index \( r_n \) ranges from 1 to \( L \), the best IoT device index \( r_n \) that maximizes the average reliability \( \Delta x_{[r_n]}^{(\beta), (l)}(x(l)) \) is searched, and the best candidate subcarrier allocation matrix \( C^{(\beta)} \) is obtained with the search result as follows.

\[
\bar{\Gamma}_{n} = \arg \max_{\Gamma_{n}} \left\{ \gamma_{[r_n]}^{[r_n]} \right\}
\]  
(41)

As is done in the techniques described above, the subcarrier allocation matrix \( C^{(\beta)} \) is updated only if the swapping at this stage increases the smallest average reliability, which is written as,

\[
C^{(\beta)}(l) = \begin{cases}
\{ C^{(\beta)}(l) \} & \gamma_{[r_n]}^{[r_n]} < \gamma_{[r_n]}^{[r_n]} \text{ otherwise}
\end{cases}
\]  
(42)

The subcarrier allocation technique proposed in the section is called “Least LLR Maximization With Channel Vector Swapping (LLRVS)”.

### 3.5 Performance Analysis of Proposed Allocation

The proposed techniques allocate the subcarriers to the IoT devices with the vector swapping or the gain swapping. The
Table 1  Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation scheme</td>
<td>QPSK / OFDM</td>
</tr>
<tr>
<td>Number of DFT points</td>
<td>128</td>
</tr>
<tr>
<td>Channel coding</td>
<td>Half rate convolutional code</td>
</tr>
<tr>
<td>Constraint length</td>
<td>3</td>
</tr>
<tr>
<td>Decoding</td>
<td>Viterbi algorithm</td>
</tr>
<tr>
<td>Number of subcarriers per device $M$</td>
<td>2</td>
</tr>
<tr>
<td>Number of transmit antennas</td>
<td>1</td>
</tr>
<tr>
<td>Number of receive antennas</td>
<td>1</td>
</tr>
<tr>
<td>Number of IoT’s devices</td>
<td>6, 10, 15</td>
</tr>
<tr>
<td>Number of subcarriers in a resource block</td>
<td>4, 5, 6</td>
</tr>
<tr>
<td>Channel model</td>
<td>Multipath fading</td>
</tr>
</tbody>
</table>

vector swapping, e.g., the CGVS, exchanges the columns in the subcarrier allocation matrix, which corresponds to the variable nodes swapping. In principle, variable node swapping does not change the shortest loop length on factor graphs. For example, variable node swapping keeps the shortest loop length of 5 on the factor graph with $N_s = 4$, $M = 2$, and $L = 6$ shown in (a) of Fig. 3.

On the other hand, the channel gain swapping, i.e., the CGGS, can change the characteristics of the factor graphs. The channel gain swapping has the possibility to find better equivalent channel matrices that make the non-orthogonal multiple access achieve better transmission performance than the vector swapping. However, the channel gain swapping possibly reduces the shortest loop length, even though the channel gain swapping maintains the number of non-zero elements in the row vectors and the column vectors of the subcarrier allocation matrix. The shortest loop with smaller loop length causes some performance degradation. For example, the channel gain swapping generates a factor graph drawn in (b) of Fig. 3. In the factor graph, the dotted lines show the two smallest loops, each of which consists of 4 edges. Since the channel gain swapping starts with a factor graph shown in (a) of Fig. 3, the channel gain swapping reduces the shortest loop length. Because it is difficult to determine whether the channel gain swapping improves the transmission performance or not, the performance of the channel gain swapping is evaluated by computer simulation in the following sections.

4. Computer Simulation

The performance of the frequency domain non-orthogonal multiple access based on the proposed subcarrier allocation is evaluated by computer simulation. The modulation scheme is quaternary phase shift keying (QPSK), and the half rate convolutional code with a constraint length of 3 is used. Multipath Rayleigh fading based on the Jakes’ model is applied to the channels between the base station and the devices. The number of the subcarriers $N_F$ and that of the
subcarriers allocated to one device $M$ are 128 and 2, respectively. The number of the subcarriers $N_s$ and that of the devices $L$ are set as $(N_s, L) = (4, 6), (5, 10),$ and $(6, 15)$, which correspond to the overloading ratios of 1.5, 2.0, and 2.5, respectively. Table 1 summarizes the simulation parameters. The performance of the fixed subcarrier allocation (FSA) is also evaluated as a reference, which is referred as the FSA in this paper. The FSA applies only one equivalent channel matrix $\bar{H}$ in spite of the channels. While any equivalent channel matrix can be applied to the FSA, for example, one of the matrices in Fig. 1 can be used for the FSA.

4.1 Comparison of Proposed Techniques

The proposed subcarrier allocation techniques are compared with each other in terms of the average bit error rate (BER) performance in Fig. 4. Horizontal axis is the $E_b/N_0$ (dB). Overloading ratio is 1.5, i.e., $(N_s, L) = (4, 6)$. Four consecutive subcarriers are allocated to one resource block, which corresponds that $b(k) = k_0 + k - 1$ where $k_0$ represents an index of the subcarrier allocated to the first device. 4-path Rayleigh fading is applied to all the channels. In the figure, the performances of the FSA and the conventional technique are added as references $^\dagger$. The proposed CGVS and the CNVS achieve about 4 dB better performance than the FSA at the BER of $10^{-4}$. They are only about 0.1 dB inferior to the conventional technique proposed in [21] at the BER of $10^{-4}$. On the other hand, the performance of the CGGS is a little bit worse than those of the other two proposed techniques, because the CGGS possibly reduces the shortest loops length on the factor graphs, which is explained in Sec. 3.5. Fig. 5 compares the LLRVS with the conventional techniques in the wireless system applied in Fig. 4. The LLRVS achieves the same performance to the conventional technique. Fig. 6 shows the BER performance of the proposed techniques in the channel with the overloading ratio of 2.0, i.e., $(N_s, L) = (5, 10)$. $N_s$ consecutive subcarriers are allocated to one resource block. 4-path Rayleigh fading is applied. While the performance of the FSA is drawn as a reference, the performance of the conventional technique and the LLRVS can’t be obtained by computer simulations due to prohibitive high complexity, which will be shown in the following section. Fig. 7 shows the BER performance of the proposed techniques in the channel with the overloading ratio of 2.5, i.e., $(N_s, L) = (6, 16)$. The channel model applied in Fig. 6 is also used. The performance of the FSA is added. However, the performance of the conventional technique and the LLRVS can’t be added due to the reason described above. The proposed techniques achieve similar performance even when the overloading ratio is increased from 1.5 to 2.5. Actually, the CGVS and the CNVS attain the highest gain of about 10 dB at the BER of $10^{-4}$ when the overloading ratio is 2.0. When the overloading ratio is raised to 2.5, the gain is reduced to about 4 dB at the BER of $10^{-4}$. In principle, the diversity gain is increased as the number of the subcarrier $N_s$ rises. Besides, as the number of the subcarrier $N_s$ is increased, the number of the signals contained in a subcarrier increases, which degrades the BER performance. When the overloading ratio is raised to 2.0, the diversity gain exceeds the performance degradation, which results in the performance improvement. When the overloading ratio is set to 2.5, the performance degradation partly offsets the diversity gain, which reduces the diversity gain. This is the reason why the proposed techniques achieve the highest gain at the overloading ratio of 2.0.

As is shown in Fig. 4 to Fig. 7, the performance of the CGVS is the same to that of the CNVS in spite of the overloading ratio, while the CGGS is a little bit inferior to them. Although the performances of the CGGS and the CNVS are of interest, the performance of the CGVS is only analyzed in detail in the following section where the overloading ratio is more than 1.5 $^{\dagger \dagger}$, because the CGVS can be imple-

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$^\dagger$ The conventional technique has been proposed in [21], which is named as “MLR”. The MLR is shown to achieve the best performance among the techniques proposed in the literature.

$^{\dagger \dagger}$ As is described above, the complexity of the conventional technique and the LLRVS grows extremely high as the overloading ratio is increased.
When the overloading ratio is raised to 2, but also the LLRVS from being evaluated by computer simulations prevents the performance of not only the conventional technique. 

\[ \Delta \] an index of the first subcarrier. The spacing \( b \) denote subcarrier spacing, the allocated subcarrier can be evaluated, interleaved subcarrier allocation is possible even if we assume that the number of the paths is increased from 2 to 8, the performance is improved by about 8 dB at the BER of \( 10^{-4} \).

On the other hand, while the \( N_s \) consecutive subcarriers are allocated to a resource block in the above performance evaluation, interleaved subcarrier allocation is possible even in wireless communication systems. For example, let \( \Delta_s \) denote subcarrier spacing, the allocated subcarrier can be defined as \( b(k - 1) = k_0 + \Delta_s(k - 1) \), where \( k_0 \) represents an index of the first subcarrier. The spacing \( \Delta_s = 0 \) means the consecutive subcarrier allocation. Fig. 9 shows the BER performance with respect to the subcarrier spacing \( \Delta_s \). As the spacing is getting bigger, higher BER performances are achieved. The increase in the spacing expands the spectrum of the resource block. In principle, even if same channel models are applied, higher diversity gain can be obtained as the spectrum is getting wider. Hence, the proposed CGVS achieves better performance as the spacing increases.

4.3 Complexity

As is shown in Fig. 5, the LLRVS achieves the same performance to the conventional technique, which is the best in all the performances of the proposed techniques. On the other hand, computational complexity has to be taken into account, when we judge which technique is suitable for wireless communication systems. The complexity of the proposed techniques are evaluated in terms of the number of multiplications and additions. Fig. 10 and Fig. 11 show the number of complex multiplications and complex additions that are needed to get the subcarrier allocation matrix converged in the proposed techniques. The CGVS, the CNVS and the CGGS have almost same complexity performances, because they determine the subcarrier allocation matrix through the MIN-MAX approach with the channel gains. Since the conventional technique executes the MPA for all the possible subcarrier allocation matrices generated by the vector swapping, the conventional technique has the highest complexity. On the other hand, the LLRVS execute the MPA only if the vector is swapped. The LLRVS can be implemented with about \( 10^{-5} \) smaller number of complex multiplications and additions than the conventional technique, when the overloading ratio is 2.0. The complexity of the CGVS, the CNVS, and the CGGS is about \( 10^{-16} \) as small as the conventional technique. Those complexity gap gets higher as the overloading ratio becomes bigger.

5. Conclusion

This paper has proposed low complexity subcarrier allocation for frequency domain non-orthogonal multiple access where many devices access with a base station. In the multiple access, a few subcarriers are allocated to each device, even if we assume that the number of the devices is more than that of the subcarriers in a resource block. The proposed subcarrier allocation techniques adaptively search for subcarrier allocation which improves the BER performance such as the average BER performance. This paper proposes 4 subcarrier allocation techniques, which are called “CGVS”, “CNVS”, “CGGS”, and “LLRVS”. While all of them seek for better subcarrier allocation based on the MIN-MAX approach, the former three techniques search nicer subcarrier allocation with only channel gains, which implements them with low computational complexity. The other technique applies the MPA to seek for better subcarrier allocation.

The proposed subcarrier allocation techniques are evaluated by computer simulation in frequency domain non-orthogonal multiple access. The proposed techniques achieve better BER performance than the fixed subcarrier allocation in spite of the overloading ratio. In non-orthogonal multiple access with overloading ratio of 1.5, the “LLRVS” achieves the same performance to the conventional technique, which has been regarded as a technique to achieve the best performance. The proposed “CGVS”, “CNVS” achieve slightly inferior performance to the LLRVS, for instance, about 0.1 dB inferior at the BER of \( 10^{-4} \) in the multiple access with the overloading ratio of 1.5. “CGVS” and “CNVS” have almost the same BER performance in spite of the overloading ratio, while “CGGS” is a little bit inferior to them. “CGVS” and “CNVS” attain a gain of about 10 dB at the BER of \( 10^{-4} \) in the multiple access with the overloading ratio of 2.0.

The complexity of the proposed techniques are also...
evaluated by computer simulation in terms of the number of the complex multiplications and the additions. While the LLRVS can be implemented with smaller complexity than the conventional technique, the complexity grows exponentially with the overloading ratio. On the other hand, the other proposed technique “CGVS”, “CNVS”, and “CGGS” can be implemented with far smaller complexity than the LLRVS. The computational complexity of the three techniques is $10^{-16}$ as small as the conventional technique when the overloading ratio is set to 2.0.

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References


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