Invited Paper

Adiabatic Quantum-Flux-Parametron: A Tutorial Review

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SUMMARY The adiabatic quantum-flux-parametron (AQFP) is an energy-efficient superconductor logic element based on the quantum flux parametron. AQFP circuits can operate with energy dissipation near the thermodynamic and quantum limits by maximizing the energy efficiency of adiabatic switching. We have established the design methodology for AQFP logic and developed various energy-efficient systems using AQFP logic, such as a low-power microprocessor, reversible computer, single-photon image sensor, and stochastic electronics. We have thus demonstrated the feasibility of the wide application of AQFP logic in future information and communications technology. In this paper, we present a tutorial review on AQFP logic to provide insights into AQFP circuit technology as an introduction to this research field. We describe the historical background, operating principle, design methodology, and recent progress of AQFP logic.

key words: QFP, adiabatic logic, low-power, energy-efficient, superconductor digital electronics.

1. Introduction

The spread of new technologies such as social networking services (SNS), artificial intelligence (AI), and the Internet of things (IoT) has led to a dramatic increase in electricity demand for information and communications technology (ICT), which is expected to reach 20% of global electricity demand by 2030 [1]. This trend indicates that a future information society, such as Society 5.0, where AI and IoT are fully exploited to collect and process huge amounts of data, will require extremely energy-efficient electronics for ICT. Therefore, many types of energy-efficient superconductor logic families have been proposed in the last decade [2–7], which have the potential to operate with less power dissipation than state-of-the-art semiconductor circuits and save energy for data processing. Furthermore, several research groups have reported successful demonstrations of large-scale, energy-efficient superconductor digital circuits [8–10], which indicates the high robustness, as well as the high energy efficiency, of superconductor logic families. Note that these demonstrations were supported by recent advances in fabrication technology for superconductor integrated circuits [11–13].

In this paper, we report a tutorial review on the adiabatic quantum-flux-parametron (AQFP) [5], which is an energy-efficient superconductor logic element based on the quantum flux parametron (QFP) [14, 15]. AQFP circuits can operate with energy dissipation near the thermodynamic and quantum limits [16] due to the energy-efficient switching process, adiabatic switching [17, 18]. We describe the historical background, operating principle, design methodology, and recent progress of AQFP logic to provide insights into AQFP circuit technology, especially for students and those who are new to this research field.

2. Historical Background

The research background of the QFP and AQFP is closely associated with the study of information thermodynamics, such as reversible computing [19, 20]. Since Landauer’s discussion on the relation between logical and thermodynamic reversibility in 1961 [21], many researchers have discussed the minimum energy dissipation in computing and the possibility of computers that can operate in a thermodynamically reversible manner (i.e., without energy dissipation). In 1970, Keys and Landauer showed that the logic state of a physical device can be switched without energy dissipation by gradually changing the potential energy shape from a single well to a double well [17], whereas the reset process dissipates some amount of energy. We refer to this switching process as adiabatic switching, referencing the terminology in adiabatic complementary metal–oxide–semiconductor (CMOS) logic [18]. In 1977, Likharev invented a superconductor device that operates via adiabatic switching, the parametric quantum (PQ) [22]. The PQ is an rf superconducting quantum interference device (SQUID) in which the critical current of the Josephson junction is modulated by applying a magnetic flux so that the potential energy shape changes.
gradually. Likharev explored reversible computing based on the PQ [23], but PQ-based circuits have never been demonstrated, for several reasons [24].

Later, Goto invented a more practical adiabatic superconductor device, the QFP [14], inspired by his own previous invention, the parametron [25]. The QFP is an rf SQUID with the Josephson junction replaced with a dc SQUID, the critical current of which can be modulated by a smaller control current than is necessary for the direct modulation of a Josephson junction. Interestingly, Likharev also proposed a similar device as a modified PQ around the same time [26]. In 1986, a five-year national project exploring the possibility of QFP-based computers, “GOTO Quantum Magneto Flux Logic [27],” started in Japan, sponsored by the ERATO program of the Research Development Corporation of Japan (JRDC). This project was led by Goto and achieved many important results [15], such as the demonstration of basic QFP circuits (e.g., shift registers [28, 29] and analog-to-digital converters [30, 31]) and studies on QFP-based reversible computing [32]. Following this project, Hitachi, Ltd. and other research groups continued research on QFP circuits [33–37]. However, in the 90s, rapid single-flux-quantum (RSFQ) logic [38] attracted more attention and became the mainstream of superconductor electronics because of its ultra-high operating speed [39].

In the 2000s, energy became the key limiter of computer performance [40–42], and energy efficiency has been the most important metric in computer design. Also, the increasing power demand for ICT indicates the need for low-power computing, as mentioned in the introduction. Therefore, we revived the QFP and proposed an energy-efficient QFP, the AQFP [5], which is designed to maximize the energy efficiency of adiabatic switching by using optimized circuit parameters and advanced fabrication processes. We have established the design methodology for AQFP logic and recently succeeded in demonstrating a 4-bit microprocessor [10]. We have also developed various systems by exploiting the physical features of the AQFP, such as reversible computers [43, 44], single-photon image sensors [45, 46], and stochastic electronics [47, 48]. This indicates the potential of AQFP logic to be widely used in future ICT.

3. Operating Principle

This section describes the operating principle of the QFP and AQFP, focusing on potential energy and adiabatic switching. We begin by deriving the potential energy of an rf SQUID, since the QFP and AQFP are based on an rf SQUID. We then derive the potential energy of the QFP and show that the QFP can perform adiabatic switching, i.e., the potential energy shape of the QFP changes gradually from a single well to a double well. Lastly, we explore the energy dissipation for adiabatic switching and show that AQFP logic operates with energy dissipation near the thermodynamic and quantum limits by maximizing the energy efficiency of adiabatic switching.

3.1 Potential energy of an rf SQUID

Figure 1 depicts a circuit diagram of an rf SQUID, which comprises a Josephson junction J and an inductor L and is biased by a magnetic flux \( \Phi_b \) provided by a bias current \( I_b \). The potential energy of the rf SQUID is given by the sum of the energy stored in \( J \) (\( U_J \)) and that in \( L \)

![Fig. 1] rf SQUID biased by a magnetic flux \( \Phi_b \).

![Fig. 2] Potential energy of an rf SQUID for \( \Phi_b = 0.5\Phi_0 \). (a) Double-well potential formed for \( \beta_L = \pi \), (b) single-well potential formed for \( \beta_L = 0.2\pi \).
(\(U_m\)).\(U_j\) and \(U_m\) are given as follows: \(U_j = -E_j \cos \phi\) and \(U_m = E_m \phi_j^2 / 2\beta_q\), where \(E_j = I_j \Phi_0 / 2\pi\) is the Josephson energy, \(\beta_q = 2\pi L_j \Phi_0\) is the normalized inductance, \(\phi\) is the phase difference across \(J\), \(\phi_q\) is the normalized flux associated with \(L\), \(I_j\) is the critical current of \(J\) and \(\Phi_0\) is the flux quantum. According to flux quantization, the relation between the phase difference and magnetic fluxes is given by \(\phi = \phi_j + \phi_q\), where \(\phi_q = 2\pi \Phi_0 / \Phi_0\) is the normalized bias flux; thus, the potential energy \(U\) of the rf SQUID is given by:

\[
\frac{U}{E_j} = \frac{U_m}{E_j} + \frac{U_j}{E_j} = \frac{(\phi - \phi_j)^2}{2\beta_q} - \cos \phi. \quad (1)
\]

This equation shows that a double-well potential is formed for \(\phi = \pi\) (i.e., \(\Phi_0 = 0.5\Phi_0\)) because \(U_j\) is maximized and \(U_m\) is minimized at \(\phi = \pi\) in \(\phi\) space, as shown in Fig. 2(a), which depicts \(U_j\), \(U_m\), and \(U\) as functions of \(\phi\) for \(\phi_0 = \pi\) and \(\beta_q = \pi\). The left-hand well is associated with a counterclockwise current in the rf SQUID and the right-hand well corresponds to a clockwise current. However, this is not always the case. As shown in Fig. 2(b), the potential energy shape is a single well for \(\beta_q = 0.2\pi\) under the same bias condition (\(\Phi_0 = \pi\)).

Importantly, a comparison between Figs. 2(a) and (b) suggests that the potential energy shape of an rf SQUID can be modulated between a single well and a double well by changing the value of \(\beta_q\), i.e., an rf SQUID can perform adiabatic switching by gradually changing \(\beta_q\). Typically, \(\beta_q\) is changed by varying \(I_c\). In the PQ, \(I_c\) is directly modulated by applying a magnetic flux to the Josephson junction. In the QFP, the Josephson junction is replaced with a dc SQUID, which works as a Josephson junction with a variable critical current.

### 3.2 Quantum flux parametron (QFP)

Figure 3 depicts a circuit diagram of the QFP, which comprises a dc SQUID (\(J_1, L_1, L_2, J_2\)) and a load inductor \(L_q\). The excitation current \(I_c\) applies a magnetic flux \(\Phi_{x1} + \Phi_{x2}\) to the dc SQUID, thereby changing its equivalent critical current. In general, the circuit parameters of the QFP are symmetrical, so that \(\Phi_x = \Phi_{x1} = \Phi_{x2}\). The input current \(I_m\) applies a magnetic flux \(\Phi_m\) to \(L_q\) and tilts the potential energy so that the QFP switches to the correct logic state. The potential energy of the QFP is given by the sum of the energy stored in \(J_1, J_2, L_1, L_2,\) and \(L_q\). The total energy stored in \(J_1\) and \(J_2\) is given by \(U_j = E_j (\cos \phi_1 - \cos \phi_2) = -2E_j \cos \phi_1 \cos \phi_2\), where \(E_j = I_j \Phi_0 / 2\pi, \phi_1 = (\phi_1 - \phi) / 2, \phi_2 = (\phi_1 + \phi) / 2, \phi_1\) and \(\phi_2\) are the phase differences across \(J_1\) and \(J_2\), respectively, and \(I_c\) is the critical current of \(J_1\) and \(J_2\). Assuming that \(L_1\) and \(L_2\) are sufficiently small, the energy stored in \(L_1\) and \(L_2\) can be neglected. The energy stored in \(L_q\) is given by \(U_q = E_L \phi_q^2 / 2\beta_q\), where \(\beta_q = 2\pi L_q / \Phi_0\) and \(\phi_q\) is the normalized flux associated with \(L_q\). According to flux quantization, the relation between the phase differences and magnetic fluxes is given by:

\[
\phi_1 - \phi_x = \phi_2 + \phi_q = \phi_q + \phi_{1m}, \quad (2)
\]

where \(\phi_q = 2\pi \Phi_0 / \Phi_0\) is the normalized excitation flux and \(\phi_m = 2\pi \Phi_0 / \Phi_0\) is the normalized input flux. Equation 2 indicates that \(\phi_1 = \phi_q\) and \(\phi_2 = \phi_q + \phi_{1m}\), so that \(U_j = -2E_j \cos \phi_1 \cos \phi_2\) and \(U_m = E_m (\phi_1 - \phi_{1m})^2 / 2\beta_q\). Consequently, \(U = U_m + U_j\) is given as follows [14]:

\[
\frac{U}{E_j} = \frac{(\phi_1 - \phi_m)^2}{2\beta_q} - 2\cos \phi_1 \cos \phi_2. \quad (3)
\]

While an rf SQUID forms a double-well potential by shifting \(U_m\) in \(\phi\) space by \(\pi\) with \(\phi_0 = \pi\), as shown in Fig. 2(a), the QFP forms a double-well potential by inverting the polarity of \(U_j\) with \(\phi_x = \pi\) (see Eq. 3). Figure 4 shows the potential energy of the QFP as a function of \(\phi_x\) for \(\beta_q = 0.4\pi\) and \(\phi_m = 0.1\pi\), which tilts the potential energy towards logic 1, while the \(\phi_x\) changes from 0 to \(\pi\). The potential energy shape gradually changes from a single well to a double well, so that the circuit state can switch to logic 1 reversibly. This switching process is referred to as adiabatic switching.
 reversible manner (i.e., without energy dissipation) in the quasi-static limit. We refer to the switching process shown in Fig. 4 as adiabatic switching [17, 18] since it becomes an adiabatic process in the quasi-static limit.

Next, we derive the potential energy of the QFP with $L_1$ and $L_2$ taken into account. Assuming that $L = L_1 = L_2$, the total energy stored in $L_1$, $L_2$, and $L_q$ is given by

$$U_m = E_0[(\phi_1^2 + \phi_2^2)/2 + \phi_1 \phi_2],$$

where $\beta_1 = 2\pi I L / \Phi_0$ and $\phi_1^2$ and $\phi_2^2$ are the normalized fluxes associated with $L_1$ and $L_2$, respectively. According to flux quantization, the relation between the phase differences and magnetic fluxes is given by:

$$\phi_1 + \phi_1^1 - \phi_2 = \phi_2 + \phi_1^2 + \phi_3 = \phi_4 + \phi_1^2.$$  

(4)

Thus, $\phi_1^1 = -[(\phi_1 - \phi_2 - \phi_0) + (\phi_2 - \phi_3)]$, and $\phi_1^2 = -[\phi_2 - \phi_3 - \phi_0]$, so that $\phi_1^2 + \phi_2^2 = 2[\phi_2 - \phi_3 - \phi_0^2] + (\phi_2 - \phi_3)^2$. Consequently, $U = U_m + U_j$ is given by:

$$U = \frac{\phi_2^2}{2\beta_2} + \frac{\phi_2^2 - \phi_2^0}{2\beta_2} + \frac{\phi_2 - \phi_3}{\beta_1} - 2\cos \phi_2 \cos \phi_3.$$  

(5)

Here we conduct the following approximation to remove $\phi_3$ from Eq. 5 [49, 50]. We assume that the third term on the right side of Eq. 5 is negligibly small since in general $\phi_2 - \phi_3$ is close to zero. Then, Eq. 5 denotes the potential energy when the load inductor $L_q$ is replaced with $L_q + L/2$. Recall that $\phi_2$ denotes the phase difference across the dc SQUID (see Eq. 3); thus, $\phi_2 - \phi_3 - \phi_0$ in the second term on the right side of Eq. 5 denotes the flux associated with $L/2$ that is connected to $L_q$ in series, and the second term represents the energy stored in $L/2$. As a result, the first and second terms on the right side of Eq. 5 can be reduced to $(\phi_2 - \phi_0^2)/(2\beta_2 + \phi_0)$ and $U$ can be approximated to [50]:

$$U \approx \frac{(\phi_2 - \phi_0^2)^2}{2\beta_2 + \phi_0}.$$  

(6)

This equation is used to estimate the potential energy shape when $L_1$ and $L_2$ are taken into account.

While we explained the operation of the QFP based on an rf SQUID since we wanted to discuss the potential energy shape, it is worth noting that the original explanation of the QFP by Goto is based on the Goto pair (i.e., the parametron using Esaki diodes) [51], which can be found in the literature [49, 52].

3.3 Adiabatic quantum-flux-parametron (AQFP)

The QFP can perform adiabatic switching without energy dissipation in the quasi-static limit; however, some amount of energy is dissipated at a finite operating speed. The switching energy $E_{sw}$ (energy dissipation per switching event) of the QFP is determined by how slowly the QFP is operated, which is quantified by the ratio between two time constants: the characteristic time of the Josephson junctions ($\tau_j$) and the duration time of the switching process ($\tau_s$). Thus, $E_{sw}$ is given as follows [16, 53]:

$$E_{sw} \approx 2l_1 \Phi_0 \tau_j \tau_s$$  

(7)

where $l_1 \Phi_0$ denotes the energy scale and coefficient 2 is required because we assume that the potential energy of the QFP is excited to a double-well potential and then reset to a single-well potential (i.e., the QFP switches twice) during a switching event. $\tau_j$ is given by the $L/R$ time constant of the Josephson junction:

$$\tau_j \approx \frac{2\pi L_1}{R} = \frac{\Phi_0}{I R} = \frac{2\pi \Phi_0 C_s}{\beta L J_c}.$$  

(8)

where $L_1 = \Phi_0/2\pi L_c$ is the Josephson inductance, $R$ is the equivalent resistance of the subgap resistance and shunt resistor (which is added to the Josephson junction in parallel to adjust the quality factor $Q$), $J_c$ is the critical current density, $C_s$ is the junction capacitance per area, $C$ is the junction capacitance, and $\beta = Q^2 = 2\pi R^2 C J_c / \Phi_0$ is the McCumber parameter [54]. $\tau_s$ is the time required for $\phi_2$ to change from 0 to $\pi$, i.e., the rising/falling time of $I_s$. If $I_s$ is sinusoidal with frequency $f$, then $\tau_s = 1/2f$.

Equations 7 and 8 can also be derived from a phase-evolution viewpoint [15, 22]. While the QFP is excited, either $J_1$ or $J_2$ switches depending on $\phi_0$. For a positive $\phi_0$, $J_1$ switches and $\phi_2$ increases by approximately $2\pi$. The energy dissipation for this excitation process ($E_x$) is given by the voltage across $J_1 (V_1)$ and $R$ with regard to $J_1$:

$$E_x = \int_0^{\tau_s} \frac{V_1}{R} dt = \int_0^{\tau_s} \left( \frac{\Phi_0^2}{2\pi} \right) dt = \int_0^{\tau_s} \frac{\phi_2}{R \tau_s} dt$$  

(9)

where $V_1 = (\Phi_0/2\pi)(d\phi_2 / dt)$, $d\phi_2 / dt \approx 2\pi / \tau_s$, and $t$ is time.

Assuming that the reset process dissipates the same amount of energy $E_r = E_x$, $E_{sw}$ is given by:

$$E_{sw} = E_x + E_r \approx \frac{2\Phi_0^2}{\tau_s}.$$

(10)

which agrees with Eqs. 7 and 8.

Equations 7 and 8 show that $E_{sw}$ decreases by increasing the value of $J_c$ and/or $\beta$. Therefore, the AQFP uses high-$J_c$ processes and high-$\beta$ (i.e., underdamped) Josephson junctions to significantly reduce energy dissipation. Figure 5 shows the simulated $E_{sw}$ of the AQFP and original QFP as a function of $\tau_c$ for $I_c = 50 \mu A$. $E_{sw}$ of the AQFP is taken from our previous work [55] and $E_{sw}$ of the original QFP is estimated from the literature [15], which shows $E_{sw} \approx 10^{-19} J$ for $I_c = 25 \mu A$, $C = 1 \mu F$, $R = 1 \Omega$, and $f = 10 \text{GHz}$ ($E_{sw}$ is normalized by $I_c = 50 \mu A$ in Fig. 5). Figure 5 shows that the AQFP operates with energy dissipation less than the original QFP by one to two orders of magnitude. Furthermore, $E_{sw}$ of the AQFP can fall below the thermodynamic limit $k_B T \ln 2$, the Landauer bound [21], by increasing $\tau_c$, where $k_B$ is the Boltzmann constant and $T$ is temperature. For
comparison, it should be noted that general logic devices, such as CMOS and RSFQ logic, operate non-adiabatically and irreversibly, thus dissipating much more energy than $k_B T$ for every switching event [56].

Here we clarify the benefit of AQFP logic by comparing its energy dissipation with that of other logic families in a simple way. The average energy dissipation per device $\overline{E_{\text{diss}}}$ of CMOS logic is given by $\overline{E_{\text{diss}}} = P/\alpha N_t$, where $P$ is power dissipation, $\alpha$ is the activity factor, $f$ is the clock frequency, and $N_t$ is transistor count. Taking Intel’s Xeon Platinum 8180 microprocessor [58] as an example, $P = 205 \text{ W}$, $f = 2.5 \text{ GHz}$, and $N_t = 8 \times 10^9$ [58, 59], which gives $\overline{E_{\text{diss}}} = 8.2 \times 10^{-17} \text{ J}$ with the assumption that $\alpha = 0.125$ [60]. This dissipation corresponds to $\sim 20,000 k_B T$ at 300 K and is far from the thermodynamic limit. For conventional superconductor logic, RSFQ, the average energy dissipation is given by $\overline{E_{\text{diss}}} = I_b V_b f \approx 0.8 I_c V_b f$, where $I_b$ and $I_c$ are the average bias and critical currents of the Josephson junctions, respectively, and $V_b$ is the bias voltage. Assuming that $I_c = 150 \mu \text{A}$, $V_b = 2.5 \text{ mV}$, and $f = 50 \text{ GHz}$, $\overline{E_{\text{diss}}} = 6.0 \times 10^{-18} \text{ J}$, which corresponds to $\sim 100,000 k_B T$ at 4.2 K. As for AQFP logic, we have demonstrated an 8-bit carry look-ahead adder with $\overline{E_{\text{diss}}} = 1.4 \times 10^{-21} \text{ J}$, or $24 k_B T$, at 5 GHz [61].

Figure 6 shows $\overline{E_{\text{diss}}}$ of CMOS, RSFQ, QFP, and AQFP logic for comparison. The solid lines in Fig. 6 denote the energy-delay product (EDP) of each logic family. The two blue solid lines are for AQFP logic; one represents the EDP based on an 8-bit carry look-ahead adder with $J_c = 10 \text{ kA/cm}^2$ [61] and the other represents the EDP based on a buffer with $J_c = 2.5 \text{ kA/cm}^2$ [53]. AQFP logic can operate with much less energy dissipation than the other logic families, and furthermore, the energy dissipation is close to the thermodynamic and quantum limits, where the quantum limit corresponds to the Planck constant. This indicates the possibility of extremely energy-efficient computing systems using AQFP logic.

We note that, in addition to energy efficiency, many other factors, such as clock speed, circuit density, and cooling overhead, should be also taken into account in actual circuit design. Therefore, it is crucial to select the best logic family and/or combine multiple logic families [46, 62] for each application, since every logic family has different advantages and disadvantages.

4. Design Methodology

This section describes how to design AQFP circuits, particularly how to power and clock AQFP circuits (i.e., four-phase clocking) and how to design basic logic gates (i.e., the minimal design). As a design example, we show the details (schematic diagram, physical layout, and numerical simulation) of an AQFP full adder to give a clear image of what AQFP circuits look like and how they work.

4.1 Clocking scheme

Special clocking schemes are required for operating AQFP circuits because all AQFP logic gates, including both combinational and sequential circuits, must be
clocked by excitation currents. Furthermore, unlike CMOS, AQFP logic gates must be clocked in the order of logic operations. Figure 7(a) shows an AQFP buffer chain powered by a typical clocking scheme, four-phase clocking [34, 63]. The entire circuit is clocked and powered by a pair of ac excitation currents \( I_{k1} \) and \( I_{k2} \) with a phase separation of 90°, which apply ac magnetic flux with an amplitude of 0.5\( \Phi_0 \) to each gate. Moreover, the dc offset current \( I_d \) applies a dc magnetic flux of \( \pm 0.5\Phi_0 \) to each gate. By doing so, logic operations are performed from phase \( \phi_1 \) to phase \( \phi_4 \) with a phase separation of 90°: the logic gates at \( \phi_1 \) and \( \phi_2 \) are clocked at the rising and falling edges of \( I_{k1} \), respectively, and those at \( \phi_4 \) and \( \phi_3 \) are clocked at the rising and falling edges of \( I_{k2} \), respectively.

Figure 7(b) depicts a circuit diagram of an AQFP buffer in the buffer chain. \( I_k \) (i.e., \( I_{k1} \) or \( I_{k2} \)) and \( I_d \) flow through the two excitation lines \( L_1 \) and \( L_2 \), respectively, thereby applying excitation flux to the dc SQUID part \( (J_1, L_1, L_2, J_2) \). Unlike the QFP shown in Fig. 3, the input current \( I_{in} \) is directly applied via \( L_{in} \), and the output current \( I_{out} \) is generated via the signal transformer, comprising \( L_q \) and \( L_{out} \) coupled to each other by \( k_{out} \). Note that the signal transformer is important for our gate design scheme, the minimal design [64]. First, the \( LI \) product of the AQFP buffer is almost fixed by \( L_q \) because the output port is separated by the signal transformer, and thus the amplitude of the signal current \( I_s \) through \( L_q \) is almost independent of what are connected to the input and output ports. Second, the signal transformer enables logical negation: the AQFP gate shown in Fig. 7(b) operates as a buffer for a positive \( k_{out} \) but operates as an inverter for a negative \( k_{out} \), i.e., a buffer and an inverter are interchangeable. These two points ensure that various logic gates can be designed by putting buffers and inverters together, as will be shown later.

Figure 8 shows simulation waveforms for an AQFP buffer chain powered by four-phase clocking with 5-GHz excitation currents \( I_{k1} \) and \( I_{k2} \). The simulation was

![Fig. 7](image)

**Fig. 7** (a) AQFP buffer chain powered by four-phase clocking. Logic operations are performed from \( \phi_1 \) to \( \phi_4 \) with a phase separation of 90°. (b) AQFP buffer. \( I_k \) and \( I_d \) apply ac and dc magnetic fluxes, respectively.

![Fig. 8](image)

**Fig. 8** Simulation waveforms for an AQFP buffer chain powered by four-phase clocking at 5 GHz. \( I_{k1} \) through \( I_{k5} \) represent the logic states of the first through fifth buffers, respectively.

![Fig. 9](image)

**Fig. 9** Logic gate design based on majority logic. (a) Majority gate. The output logic \( q \), i.e., the polarity of \( I_{out} \), is determined by the majority vote of \( I_s \), \( I_{in} \), and \( I_d \). (b) AND gate. (c) NOR gate.

![Fig. 10](image)

**Fig. 10** AND cell for the HSTP, which comprises two buffer cells (20 μm by 40 μm each), a constant-0 cell (20 μm by 40 μm), and a branch cell (60 μm by 25 μm). The cell dimensions are 60 μm by 65 μm.
conducted using a Josephson circuit simulator, JSIM [65], and the circuit parameters used in the simulation are based on a recent cell design \((I_c = 50 \mu A)\) [66]. \(I_n\) is the input current applied to the first buffer. \(I_1\) through \(I_3\), which correspond to \(I_s\) in Fig. 7(b), are the signal currents that represent the logic states of the first through fifth buffers, respectively. The polarity of \(I_s\) represents the logic state of a buffer: a positive \(I_s\) denotes the logic-1 state and a negative \(I_s\) denotes the logic-0 state. Thus, Fig. 8 shows that data stream “01010011” propagates through the buffer chain with a latency of a quarter clock cycle (50 ps at 5 GHz) per gate. This latency is not sufficiently small for some applications; thus, we have also proposed a low-latency clocking scheme, delay-line clocking [67].

4.2 Logic gate design

AQFP logic gates are designed based on majority logic [14, 25], since logical values are represented by the polarity of currents. Figure 9 shows three examples of combinational circuit design. Figure 9(a) depicts a three-input majority (MAJ) gate comprising three buffers. The output currents from the three AQFP buffers \((I_a, I_b, \text{ and } I_c)\) are merged at the output port, so that the polarity of the output current \(I_o\) is determined by whether the majority is positive or negative among \(I_a, I_b, \text{ and } I_c\), assuming that the amplitude is almost the same among \(I_a, I_b, \text{ and } I_c\) due to the signal transformer in each buffer. Thus, the output \(q\) is given by \(q = MAJ(a, b, c) = ab + bc + ca\), where \(a, b, \text{ and } c\) are the inputs. Furthermore, various logic gates can be designed based on the MAJ gate because majority logic includes both logical conjunction and disjunction. For instance, by fixing input \(b\) to 0, the MAJ gate becomes an AND gate that conducts \(q = ac\), as shown in Fig. 9(b). Moreover, since buffers and inverters are interchangeable in AQFP logic, a NOR gate can be designed by replacing two buffers in the AND gate with two inverters [i.e., \(q = \overline{a} \cdot \overline{c} = a + c\)], as shown in Fig. 9(c). As for sequential circuits, a flip-flop can be designed by making a feedback line [68] or storage loop [69] in an AQFP gate.

The three examples shown in Fig. 9 suggest that basic logic gates can be formed from several common components. Therefore, in the development of AQFP cell libraries, the minimal design [64] is adopted so that various logic cells can be quickly designed based on majority logic. In the minimal design, four types of building-block cells (buffer, inverter, constant, and branch cells) are designed in advance, and then other logic cells are designed by putting the building-block cells together like interlocking toy bricks. A constant cell [70] is a gate that generates a constant 0 or 1 signal and a branch cell consists of inductor branches for merging or dividing currents. As an example, we show the layout design of the AND cell for the AIST 10 kA/cm² Nb high-speed standard process (HSTP) [63] in Fig. 10. First, each building-block cell is carefully designed using the three-dimensional parameter extractor, InductEx [71, 72]. Then, the AND cell is designed by putting together two buffer cells, a constant-0 cell, and a branch cell, where the constant-0 cell generates a 0 signal and the branch cell merges the output currents from the buffer and constant cells. Likewise, many other logic cells can be designed by putting building-block cells together, without parameter or layout optimization; thus, AQFP cell libraries can be quickly developed and tailored for different fabrication processes [63, 64].

4.3 Circuit design example

Figure 11(a) depicts a circuit diagram of a full adder comprising three MAJ cells [25]. Additional buffers are inserted for phase adjustment and fanout; the output current from a MAJ cell can be small due to the interaction between the three buffers in the MAJ cell,
and thus the fanout of a MAJ cell (also, an AND cell, OR cell, and other similar cells) is limited to one in our design rule. Figure 11(b) shows the layout design of the full adder for the HSTP. The AQFP logic cells are placed along the meandered excitation lines, through which the excitation currents $I_{x1}$ and $I_{x2}$ flow. Since the frequency of $I_{x1}$ and $I_{x2}$ is equal to the clock frequency, microwave excitation currents flow through the excitation lines at high clock frequencies. Hence, the impedance of the excitation lines, which are 50 Ω microstrip lines, are carefully designed using InductEx [66, 73] so that standing waves do not appear along the excitation lines. Figure 11(c) shows simulation waveforms for the AQFP full adder at 5 GHz, where $I_{in}$, $I_{inb}$, and $I_{inc}$ are the input currents, and $I_{carry}$ and $I_{sum}$ are the signal currents of the buffers placed after the full adder. This figure shows that the carry and sum signals are generated with a latency of four phases, i.e., one clock cycle.

5. Recent Progress

We have established design environment and tools for AQFP logic [74], such as hardware description language-based digital simulation [75], logic synthesis [76], routing and placement tools [77], and the energy estimation method [78], and demonstrated large-scale AQFP circuits with ~1,000 to ~20,000 Josephson junctions. Furthermore, we have developed various systems that exploit the physical features of AQFP logic. This section describes the recent progress on AQFP system development.

5.1 Low-power microprocessor

Since AQFP logic exhibits extremely high energy efficiency, the most straightforward application is the development of low-power microprocessors. We have designed and demonstrated datapath components, such as adders [61, 79], register files [80, 81], and an arithmetic and logic unit (ALU) [74]. Moreover, we have developed an AQFP microprocessor called Monolithic Adiabatic Integration Architecture (MANA) [10]. This processor adopts a reduced instruction set computer (RISC) architecture with 4-bit data words via 16-bit instruction words. Figure 12(a) shows a micrograph of the MANA chip that was fabricated using the HSTP, which includes 21,460 Josephson junctions. We have succeeded in demonstrating test programs at a low clock frequency, which include read/write of the register file, ALU execution, branching, and hardware stalling. The estimated energy dissipation of MANA is $3.0 \times 10^{-17}$ J per operation. The above results indicate that large-scale, energy-efficient computing systems can be achieved using AQFP logic. Furthermore, we plan to build a more practical MANA by adopting the double-gate process [82] and directly coupled QFP logic [83] for higher circuit density, delay-line clocking [67] for reducing latency, and the high-speed voltage driver [84] and multi-block clock distribution [81] for high-frequency operation.

5.2 Reversible computer

A reversible computer [19, 20] is an ideal computer that can operate in a thermodynamically reversible manner, i.e., without energy dissipation, in the quasi-static limit. Many researchers have proposed physical models for building reversible computers [7, 20, 23, 32, 85–89]. We have been investigating reversible computing using AQFP logic since the AQFP is an intrinsically reversible device (based on the fact that adiabatic switching is a reversible process). We have proposed a reversible logic gate based on the AQFP, the reversible quantum-flux-parametrón (RQFP) [43, 44]. The RQFP comprises six AQFP gates placed and interconnected symmetrically, and is logically and physically reversible due to the bijective truth table and symmetrical structure. In a numerical simulation, we showed that the RQFP can perform logic operations in a thermodynamically reversible manner (i.e., reversible computing) in the
quasi-static limit [44]. Furthermore, we have fabricated and demonstrated an RQFP gate [43] and RQFP-based reversible full adder [90]. These demonstrations indicate that the RQFP is a practical reversible logic gate and can be used as a building block for making reversible computers. We also verified Landauer’s principle [21] in a numerical simulation to reveal the energy dissipation for data erasure [91]. It is noteworthy that whereas a single AQFP can operate reversibly via adiabatic switching, conventional AQFP logic gates, such as MAJ, AND, and OR gates, operate irreversibly and dissipate energy via non-adiabatic processes due to logical and physical irreversibility [44], which causes an inappropriate interaction between AQFPs and lead to non-adiabatic operations. Therefore, there is room for improvement in the energy efficiency of AQFP logic by adopting reversible computing. We have been developing a reversible microprocessor using the RQFP, which has the potential to operate with much less energy dissipation than MANA.

5.3 Single-photon image sensor

AQFP logic exhibits physical features that are suitable for detector applications, namely low-current drive and high sensitivity. First, the amount of supply currents for driving an AQFP circuit (typically, a few milliamps) does not increase with the circuit scale because many logic gates can be coupled to a few common excitation lines, as shown in Fig. 11(a). We have demonstrated an AQFP circuit with approximately 20,000 Josephson junctions using excitation currents of 4.8 mA [92]. Low-current drive is important for cryocooler implementation because the amount of supply currents is limited by the cooling power. Second, AQFP logic has high sensitivity because in adiabatic switching the circuit state can switch to the correct logic state by slightly tilting the potential energy, as shown in Fig. 4. We have demonstrated a high-sensitivity AQFP comparator with a sensitivity of 46 nA [93]. High-sensitivity circuits enable the analog-to-digital conversion of tiny signal currents from cryogenic detectors, such as superconducting nanowire single-photon detectors (SSPDs or SNSPDs) [94, 95], inside a cryocooler. Therefore, we proposed using AQFP logic as readout circuits for SSPD arrays to reduce the number of cables required for reading out the SSPD pixels [45], towards the development of single-photon image sensors. We demonstrated a single-pixel SSPD implemented with an AQFP comparator in a 0.1-W Gifford-McMahon (GM) cryocooler [96] to verify that AQFP logic has sufficient sensitivity to sample the signal currents from an SSPD. Furthermore, we proposed a readout circuit for large-scale SSPD arrays, the AQFP/RSFQ hybrid interface [46], which combines AQFP and RSFQ logic families to achieve both high scalability and high time resolution. Figure 12(b) shows a micrograph of a hybrid interface fabricated using the AIST 2.5 kA/cm² Nb standard process (STP2) [97]. As a proof of concept, we demonstrated a four-pixel SSPD array using the hybrid interface and found that each pixel was read out with low error rates and low timing jitter [46]. The next step is to develop hybrid interfaces for large-scale SSPD arrays.

5.4 Stochastic electronics

AQFP logic can perform stochastic operations using thermal fluctuations [91]. Imagine a potential energy shape during adiabatic switching (Fig. 4) for $\phi_n = 0$; in this case, the potential energy shape is always symmetrical during excitation. Hence, the circuit state switches to either logic 0 or 1 stochastically due to thermal fluctuations. This indicates that the probability distribution of the AQFP can be easily controlled by the input current. Therefore, we have developed AQFP-based stochastic electronics. We proposed a random number generator (RNG) based on an AQFP buffer and demonstrated high-quality random bit streams [48]; a similar RNG [98] was also proposed using a negative-inductance SQUID [85]. The AQFP RNG may be used for stochastic computing-based neural networks [99]. Furthermore, we proposed using AQFP logic to implement stochastic local search algorithms [47]. Specifically, we developed an amoeba-inspired problem solver (APS) [100, 101] using AQFP logic and demonstrated that the AQFP-based APS can find solutions to a simple logical constraint satisfaction problem using thermal fluctuations [47]. Figure 12(c) shows a micrograph of the AQFP-based APS. While this APS can solve only one instance with four variables, we plan to develop large-scale, practical APSs based on AQFP field-programmable gate arrays [62].

6. Conclusion

We reported a tutorial review on AQFP logic. Following a brief introduction of the historical background, we explained the operating principle of the QFP and AQFP, with a particular focus on the potential energy. We then described how to design AQFP circuits and detailed recent progress on AQFP system development. We hope that this tutorial review will provide insights into AQFP logic for those who are new to this research field.

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