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Analyses of Transient Energy Deposition in Biological Bodies Exposed to Electromagnetic Pulses Using Parameter Extraction Method

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SUMMARY In this study, we develop a numerical method for determining transient energy deposition in biological bodies exposed to electromagnetic (EM) pulses. We use a newly developed frequency-dependent finite-difference time-domain (FDTD) method, which is combined with the fast inverse Laplace transform (FILT) and Prony method. The FILT and Prony method are utilized to transform the Cole–Cole model of biological media into a sum of multiple Debye relaxation terms. Parameters of Debye terms are then extracted by comparison with the time-domain impulse responses. The extracted parameters are used in an FDTD formulation, which is derived using the auxiliary differential equation method, and transient energy deposition into a biological medium is calculated by the equivalent circuit method. The validity of our proposed method is demonstrated by comparing numerical results and those derived from an analytical method. Finally, transient energy deposition into human heads of TARO and HANAKO models is then calculated using the proposed method and, physical insights into pulse exposures of the human heads are provided.

key words: specific energy absorption, Cole–Cole model, finite-difference time-domain, electromagnetic pulse.

1. Introduction

There are many applications of ultra-wideband (UWB) electromagnetic pulses (EMPs) including radars for military and commercial autonomous vehicles [1], non-destructive evaluation [2, 3], human vital signal detection [4], in-body/on-body communications [5, 6], and medical inspection and treatment [7, 8]. Despite the increasing use of EMPs, there are insufficient information and understanding of aspects of interactions between biological bodies and EMPs owing to the complexity in treating wide spectrum components of the EMPs and difficulties in treating the frequency dependence of the dielectric properties of biological media.

Nevertheless, the interactions between biological bodies and EMPs have been extensively investigated by numerical simulations and experiments. Recognized biological effects due to EMPs include the microwave hearing effect [9, 10], microwave heating effect [11, 12], and electroproportion [13]. Microwave hearing is the phenomenon of perceiving an audible buzz or chirp sound as a result of energy absorption of microwave pulses by soft tissues inside the head [14], while the microwave heating effect is caused by the transformation of absorbed EM energy into heat. Note that the amount of absorbed energy may vary according to the location, tissue type, and geometrical factors of biological bodies, and hence it usually exhibits complex distributions depending on the exposed subject. Meanwhile, electroproportion raises concern about the breakdown of lipid bilayer membranes at the cell level [15] and it may be excluded when considering macroscopic EM phenomena.

To understand these phenomena correctly, distributions of the absorbed energy or induced EM fields inside biological bodies is essential.

Distributions of the absorbed energy and induced EM fields are governed by permittivity, conductivity, incident source configuration, and geometrical factors that describe the biological structure. In the literature, canonical shapes such as multilayer tissues, dielectric spheres, and prolate spheroidal models representing biological bodies were used as targets for the earliest investigation on interactions between EMPs and biological systems [16–18]. These studies derived the EM distribution at frequencies of interest, yet there is lack of information about the EM waveform inside the targets except in a few references, to the best of the authors’ knowledge [19]. The main problem in deriving the pulse waveform inside biological bodies is due to the difficulty in incorporating the frequency dependence of the dielectric properties of the biological tissues, which is represented by the four-term Cole–Cole model [20]. Some studies adopted the fast Fourier transform or fast inverse Laplace transform (FILT) [21] in the calculation of the exact time-domain waveforms but a new calculation is required at every observation point inside biological bodies [22, 23]. As a result, large computational resources are needed in obtaining solutions across the whole analysis region.

To calculate the energy and EM distributions inside biological bodies, the finite-difference time-domain (FDTD) method has received much attention since it was proposed in 1966 [24]. This method uses a staggered grid model and adopts a leapfrog algorithm in advancing electric and magnetic field values at every point in the analysis space. Therefore, the induced current or specific absorption rate...
(SAR) inside a detailed human body can be found at once after the simulations ended. The popularity of the FDTD method is due to its capability to treat heterogeneous models in a straightforward manner and its high computation efficiency, as well as its high degree of affinity with large-scale parallelization using supercomputers. \[25, 26\]. To evaluate exposure from a short pulse, many frequency-dependent finite-difference time-domain (FDTD) methods have been developed and used \[27-30\]. The methods use the first- or second-order Debye relaxation equation to express the complex permittivity of the medium. However, as shown in \[31\], even fourth-order Debye terms cannot provide sufficiently accurate and valid results for the transient EM distribution inside biological bodies. Recently, an alternative approaches using the Padé approximant or Riemann–Liouville theory to formulate the Cole–Cole relaxations have been proposed \[32, 33\]. The authors have also recently proposed a numerical approach combining the FILT and Prony method to formulate the Cole–Cole model. The proposed method has been successfully used to determine the specific energy absorption inside a numerical human head due to a single pulse \[31, 34\]. It has also been applied to design a UWB antenna for use in the vicinity of the human body \[35\].

In this paper, we develop a new FD2TD approach, in which multiple Debye parameters extracted from the Cole–Cole model are incorporated with the aid of the FILT and the Prony method, for calculation of the transient energy deposition into biological bodies. Multiple Debye terms are formulated into the FDTD update equation by applying the auxiliary differential equation (ADE) method. The paper is formulated into the FDTD update equation by applying the

\[ F_r(\omega) = \varepsilon_{\infty} + \frac{\sigma}{j\omega\varepsilon_0} + \sum_{i=1}^{4} \chi_i(\omega) \]  

(1)

where

\[ \chi_i(\omega) = \frac{\Delta \chi_i}{1 + (j\omega \tau_i)^{1-\alpha_i}} \]  

(2)

represents the electric susceptibility of the \(i\)th relaxation. \(\varepsilon_{\infty}\)

and \(\sigma\) are the relative permittivity at an infinite frequency and the conductivity, respectively. \(\varepsilon_0\) is the free-space permittivity. \(\Delta \chi_i\) denotes the change in the relative permittivity due to the \(i\)th relaxation. \(\tau_i\) is the characteristic relaxation time, and \(\alpha_i\) is a parameter determining the broadness of the \(i\)th relaxation. To extract Debye parameters of the Cole–Cole model, we first modify the frequency-domain electric susceptibility to that in the complex frequency domain by substituting \(j\omega\) with the complex frequency variable \(s\). Then it is numerically transformed into the time-domain impulse response by the FILT \[21, 36\]. The parameter of the impulse response in the \(z\)-domain is finally extracted using the Prony method. For the Cole–Cole relaxation, it was found that the electric susceptibility represented by (2) can be written as a sum of the Debye relaxations, which can be expressed in the \(z\)-domain as

\[ x(z) = \sum_{i=1}^{N_i} \frac{A^{(i)}_p}{1 - p^{(i)}_t z^{-1}} \]  

(3)

where \(N_i\) is the number of real poles for the \(i\)th relaxation \((i = 1, 2, 3, \text{or } 4)\). Note that the number of real poles may be different for each relaxation term. \(A^{(i)}_p\) and \(p^{(i)}_t\) are the coefficients for each Debye pole determined directly from partial fraction expansion of the \(z\)-domain impulse responses. Note that (3) is a monotonically decreasing function in the time domain and, therefore, may be expressed by a sum of Debye terms. The time-domain impulse responses of the Cole–Cole model are determined as

\[ x(t) = \sum_{i=1}^{N_i} \Delta x^{(i)}_t \exp\left(-\frac{t}{\tau^{(i)}_t}\right) \]  

(4)

where \(\Delta x^{(i)}_t\) and \(\tau^{(i)}_t\) are the Debye parameters which are related to the coefficients \(A^{(i)}_p\) and \(p^{(i)}_t\) as follows \[37\]:

\[ \tau^{(i)}_t = -\Delta t \frac{\ln p^{(i)}_t}{A^{(i)}_p} \]  

(5)

\[ \Delta x^{(i)}_t = \frac{A^{(i)}_p}{\tau^{(i)}_t} \]  

(6)

where \(\Delta t\) denotes the time step interval used in the normalization of the relaxation time \(\tau_t\) when applying the FILT to (2). The expression in the frequency domain is then determined as

\[ x(\omega) = \sum_{i=1}^{N_i} \Delta x^{(i)}_t \frac{1}{1 + j\omega \tau^{(i)}_t} \]  

(7)

The Cole–Cole model is now transformed into a sum of Debye models in the frequency domain. It should be noted that (7) can now be formulated using conventional recursive convolution (RC) techniques, ADE method, or the \(z\)-transformation and the time step interval in the FDTD formulation can now be chosen arbitrarily.
2.2 Formulation of Frequency-Dependent FDTD Method Using Auxiliary Differential Equation Method

Electromagnetic fields in isotropic media are described in the frequency domain by Maxwell’s equations as

\[
\nabla \times \mathbf{H}(\omega) = j \omega \varepsilon_0 \mathbf{E}(\omega)
\]

\[
\nabla \times \mathbf{E}(\omega) = -j \omega \mu_0 \mathbf{H}(\omega)
\]

where \(\mathbf{E}(\omega)\) and \(\mathbf{H}(\omega)\) are the time-harmonic electric and magnetic fields, respectively. \(\mu_0\) is the permeability, \(\varepsilon_0\) is the permittivity in free space, and \(\varepsilon_\text{r}(\omega)\) is the complex relative permittivity, which is expressed as (1) for biological tissues. Substituting (1) into (8) gives

\[
\nabla \times \mathbf{H}(\omega) = j \omega \varepsilon_0 \left( \varepsilon_{\text{r}_0} + \frac{\sigma}{j \omega \varepsilon_0} \right) \mathbf{E}(\omega)
\]

(10)

Each Cole–Cole term can be transformed into multiple Debye terms as (7), therefore, (10) is expressed as

\[
\nabla \times \mathbf{H}(\omega) = j \omega \varepsilon_0 \varepsilon_{\text{r}_0} \mathbf{E}(\omega) + \frac{\sigma}{j \omega \varepsilon_0} \mathbf{E}(\omega) + \sum_{i=1}^{N_i} \mathbf{J}_i(\omega)
\]

(11)

where the nested sums of Debye terms are merged into a single summation, \(N_i = N_1 + N_2 + N_3 + N_4\) is the total number of Debye terms contributed from the four Cole–Cole relaxation terms, and

\[
\mathbf{J}_i(\omega) = \frac{j \omega \varepsilon_0 \Delta \varepsilon_i}{1 + j \omega \tau_i} \mathbf{E}(\omega)
\]

(12)

represents the polarization current density for each Debye term. By taking the inverse Fourier transform of (12), we obtain the differential equation describing the relationship between \(\mathbf{E}\) and \(\mathbf{J}\) as

\[
\tau_i \frac{\partial \mathbf{J}_i(t)}{\partial t} + \mathbf{J}_i(t) = \varepsilon_{\text{r}_0} \Delta \varepsilon_i \frac{\partial \mathbf{E}(\omega)}{\partial t}
\]

(13)

The above equation is then discretized using the central difference method at the time step \(t = (n-1/2)\Delta t\) and the update equation for \(\mathbf{J}\) is obtained as

\[
\mathbf{J}_i^n = q_i \mathbf{J}_i^{n-1} + B_i \left( \mathbf{E}^n - \mathbf{E}^{n-1} \right)
\]

(14)

where

\[
q_i = 1 - \frac{\Delta t}{2 \tau_i} B_i = \frac{\varepsilon_{\text{r}_0} \Delta \varepsilon_i}{\tau_i} \left( 1 + \frac{\Delta t}{2 \tau_i} \right)
\]

(15)

Transforming (11) into its time-domain expression and applying the central difference at the same time step \(t = (n-1/2)\Delta t\) gives

\[
\frac{\Delta \mathbf{E}^n}{\varepsilon_0 \Delta t} \nabla \times \frac{\mathbf{H}^n}{\varepsilon_0^{n-1/2}} = \left( \varepsilon_{\text{r}_0} + \frac{\sigma \Delta t}{2 \varepsilon_0} \right) \mathbf{E}^n - \left( \varepsilon_{\text{r}_0} - \frac{\sigma \Delta t}{2 \varepsilon_0} \right) \mathbf{E}^{n-1} + \frac{\Delta t}{\varepsilon_0} \sum_{i=1}^{N_i} \mathbf{J}_i^n + |\mathbf{J}_i^{n-1}| \frac{2}{2}
\]

(16)

By substituting (14) into (16), the update equation for the electric field is formulated as

\[
\mathbf{J}_i = \frac{1}{K_0} \left[ G_0 (1 + q_i) \mathbf{J}_i^{n-1} + \Delta \mathbf{E} \times \mathbf{H}^{n+1/2} \right]
\]

(17)

where,

\[
\mathbf{J}_i^n = q_i \mathbf{J}_i^{n-1} + B_i \left( \mathbf{E}^n - \mathbf{E}^{n-1} \right)
\]

(18)

\[
K_0 = \varepsilon_{\text{r}_0} + \frac{\sigma \Delta t}{2 \varepsilon_0} + \sum_{i=1}^{N_i} B_i
\]

(19)

\[
G_0 = \varepsilon_{\text{r}_0} - \frac{\sigma \Delta t}{2 \varepsilon_0} + \sum_{i=1}^{N_i} B_i
\]

(20)

\[
B_i = \frac{\Delta \varepsilon_i \Delta t}{\tau_i^2 + \Delta \varepsilon_i}
\]

(21)

Note that \(\mathbf{J}_i = \Delta t/(2 \varepsilon_0) \mathbf{J}_i^n\) is applied here to simplify the formulation.

2.3 Calculation of Specific Energy Absorption

The terms used in this study for different physical quantities are defined as follows:

\(S\) : Specific energy absorption in J/kg, representing the cumulative amount of energy absorbed into a tissue per unit mass [38].

\(S_{\text{electric}}\) : Specific energy loss in J/kg, representing the per-mass cumulative energy loss through dissipation into a tissue, which is transformed into heat [39]

\(P_E\) : Transient electric power density in W/m³, which indicates the density of electric energy into a tissue per unit time. Note that the electric energy may exhibit a negative value when an excessive release of energy occurs.

\(P_L\) : Transient power loss density in W/m³, which represents the power dissipated into a tissue per unit volume [40]. Note that only a positive value is allowed for \(P_L\) since it represents a real loss in the medium.

Firstly, \(S_A\) is calculated using FDTD simulations with the following equations [31, 38]:

\[
S_A = \frac{1}{\rho} \int_{0}^{t_{\text{max}}} \mathbf{E}(t) \cdot \frac{\partial \mathbf{D}(t)}{\partial t} dt \approx \frac{\Delta \varepsilon}{\rho} \sum_{n=1}^{N} \left( \mathbf{P}_E^n - \mathbf{P}_E^{n-1/2} \right)
\]

(22)

where

\[
\mathbf{P}_E^{n-1/2} = \frac{1}{2 \Delta t} \left( \mathbf{E}^n + \mathbf{E}^{n-1} \right) \cdot \left( \mathbf{D}^n - \mathbf{D}^{n-1} \right)
\]

and \(\rho\) is the mass density of biological tissues. As the explicit physical interpretation, \(P_E\) represents the sum of two physically distinct quantities: one that represents the power density dissipated into the medium in the form of heat and one that presents the sum of the power density stored in the field and the reactively stored power density in the medium [41]. Therefore, \(P_E\) may exhibit a negative value due to an excessive release of the stored energy at a certain time. However, when the reactive electric energy decreases over a long calculation time after the EM pulse vanishes, \(S_A\) will
eventually reach the same value as the real energy loss $S_{AL}$ as will be shown later in our numerical results. Meanwhile, $P_l$ represents only the power density loss in the medium which is transformed into heat. To derive $S_{AL}$ and $P_l$ in a medium, we employ the equivalent circuit (EC) approach in the FDTD formulation as in [39, 42]. Since (11) can be considered as an EC in Fig. 1, the electrical parameter values per unit length of the lumped elements are derived as

$$G_0 = \sigma, C_0 = \varepsilon_0 \varepsilon_{\infty}$$ (24)

$$R_l = \frac{\tau_l}{\varepsilon_0 \Delta \chi_l}, G_l = \varepsilon_0 \Delta \chi_l$$ (25)

The total current density in the time-domain is expressed as

$$J(t) = J_c(t) + J_p(t) + \sum_{i=1}^{N} J_i(t)$$ (26)

where $J_c(t) = C_0 \frac{\partial E(t)}{\partial t}$, $J_p(t) = G_0 E(t)$, and $J_i(t)$ is the current density flowing into the RC branches as shown in Fig. 1. Each branch corresponds to a Debye relaxation term whose parameters are extracted using the proposed parameter extraction method described in Section 2.1. Therefore, the power loss densities for the lumped resistances $R_l$ and parallel conductance $G_0$ in Fig. 1 are calculated using

$$P_{R_l}(t) = R_l J_c(t) \cdot J_c(t)$$ (27)

$$P_{G_0}(t) = \frac{1}{G_0} J_c(t) \cdot J_c(t).$$ (28)

The power loss density due to the lumped resistances $R_l$ and parallel conductance $G_0$ at a discrete time can be determined as

$$p^n_{R_l} = \frac{\tau_l}{\varepsilon_0 \Delta \chi_l} J^n_c \cdot (J^n_c)^* = \frac{\Delta \chi_l}{\Delta t} |J^n_c|^2$$ (29)

$$p^n_{G_0} = \sigma |E^n|^2$$ (30)

respectively. Finally, $S_{AL}$ is expressed as

$$S_{AL} = \int_0^{\tau_{max}} \left[ P_{G_0}(t) + P_{R_l}(t) \right] dt \approx \frac{\Delta t}{\rho} \sum_{n=1}^{N_{HT}} p^{n-1/2}_l$$ (31)

where

$$p^{n-1/2}_l = \sigma |E^{n-1/2}|^2 + \frac{1}{\varepsilon_0 \Delta \chi_l} \sum_{i=1}^{N} \frac{\tau_i}{\varepsilon_0 \Delta \chi_l} |J^{n-1/2}|^2$$ (32)

represents the transient specific power loss density at the $(n-1/2)$th time step. Note that the electric field and current density at the $(n-1/2)$th time step are determined by averaging of those at the $n$th and $(n-1)$th time steps. $S_{AL}$ is the only term to be considered when calculating the temperature rise since it represents the absorbed energy that is transformed into heat. The SAR in W/kg at a discrete time step can also be calculated from $S_{AL}$ as

$$\frac{\partial S_{AL}(t,t)}{\partial t} \bigg|_{t=(n-1/2)\Delta t} \approx \frac{p^{n-1/2}_l}{\rho} = \text{SAR}^{n-1/2}.$$ (33)

3. Numerical Validation

3.1 Determination of Debye Parameters from Cole–Cole Model

To begin with, we investigate the validity of the proposed formulations for Debye relaxation expressed by (2) having parameters of $\Delta \chi_l = 3.0$, $n = 64$ ps, and $\alpha_l = 0$. Time step interval is set to $\Delta t = 3.85$ ps. After applying the FILT to (2) with the relaxation time normalized by $\Delta t$, the impulse response sequence is determined at $n = 0.5, 1.5, 2.5, \ldots, 4000.5$ since we cannot directly obtain the result at $n = 0$ using the FILT. The maximum number of terms required to satisfy an accuracy of $10^{-5}$ in the time-domain impulse response is 1179 when $n = 4000.5$. The Prony method is then applied to derive the coefficients $A_l$ and $p_l$ with an allowable error of $10^{-5}$. Note that the extracted coefficients $A_l$ and $p_l$ are for $n = 0.5$. However, $A_l$ and $p_l$ at $n = 0$ are required for the parameter extraction to determine the Debye parameters by (5) and (6). Here we use the following equations to approximate the coefficients $A_l$ and $p_l$ at $n = 0$:

$$A_l^{(0)} = \frac{\sqrt{|p_l^{(0)}|}}{A_l^{(0.5)}}, P_l^{(0)} = p_l^{(0.5)}$$ (34)

where $A_l^{(0.5)}$ and $p_l^{(0.5)}$ are the residues and poles derived from the impulse response sequence at $n = 0.5, 1.5, 2.5, \ldots, 4000.5$, respectively. The superscript indicates the start time step index of the impulse response sequence which is $n = 0.5$ in our study. Using (5) and (6), we then obtain $\Delta \chi_l = 2.999999$, $\alpha_l = 64.0025$ ps, which are almost identical to the exact values of $\Delta \chi_l = 3.0$, $\alpha_l = 64$ ps. Therefore, the validity of our extraction method has been demonstrated.

Next, we adopt our Debye parameter extraction method to the Cole–Cole model of muscle. The Cole–Cole parameters are extracted from Gabriel’s database [20]. Using the same procedures as above, we obtain the residues $A_l$ and poles $p_l$ as tabulated in Table I. Numbers of coefficients are determined to be 8, 12, and 12 for the first, second, and third Cole–Cole relaxation terms, respectively, with an allowable error of $10^{-5}$. The extracted parameters are also shown in Table I and plotted in Fig. 2, along with their theoretical values in solid black lines, which are determined from (7) of [43]. Each point indicates the relaxation frequency and corresponding $\Delta \chi_l$ of the Debye model in (7). The relaxation frequency is calculated using $f_l = 1/\tau_l$, where $\tau_l$ denotes the relaxation time of each Debye term. The parameters extracted using our proposed method are in fair agreement with the theoretical values. After the parameters are
where the reflection coefficient of each Cole–Cole term is calculated using the following equation:

\[ \alpha_{\omega} = \frac{\epsilon_{\omega} - 1 + \chi_{t}(\omega)}{\epsilon_{\omega} + 1 + \chi_{t}(\omega)} \]  

(36)

where \( \chi_{t} \) is the Cole-Cole electric susceptibility expressed in (2). The results for all Cole–Cole terms show good agreement with theoretical values, confirming the validity of the method. Errors in the reflection coefficients when compared with theoretical values are also calculated and depicted as a solid black line in Fig. 4. The errors for the proposed method are less than 0.3% over a broad frequency range from 10 MHz to 1 GHz. The errors match those derived by the method in [31] up to around 100 MHz whereas we can see an increase in the errors at higher frequencies over 200 MHz. The increase may be due to the difference in the update equation of the auxiliary current that is 33 when an allowable error of 10^{-5} is chosen. An EM sphere is coincides with the center of the sphere. The radius of the sphere of the muscle tissue. The origin of the coordinates with analytical results obtained from Mie theory [45]. Fig. 5 shows the reflection coefficients obtained from theory and numerically derived using the extracted Debye parameters for each Cole–Cole term.

3.2 Energy Deposition into Homogeneous Dielectric Sphere

To demonstrate the validity of the proposed FDTD scheme, numerical analyses of the electric field waveform and the specific power loss dissipated into a homogeneous dielectric sphere having dielectric properties represented by four Cole–Cole relaxation terms are calculated and compared with analytical results obtained from Mie theory [45]. Fig. 5 illustrates the analysis model of the homogeneous dielectric sphere of the muscle tissue. The origin of the coordinates coincides with the center of the sphere. The radius of the sphere is \( r_s = 0.1 \) m. The number of coefficients for muscle is 33 when an allowable error of 10^{-5} is chosen. An EM

\[
\epsilon_{\omega}(t) = \exp \left(-\frac{(t - T_0)^2}{\alpha_0^2} \right),
\]

(35)

where \( T_0 = 0.385 \) ns and \( \alpha_0 = 0.194 \) ns, is used as a source. The applied pulse contains broad frequency components from dc to approximately 5 GHz. Fig. 3 is an example of the electric field waveform as a function of time for a homogeneous sphere having dielectric properties represented by four Cole–Cole relaxation terms.

TABLE I

<table>
<thead>
<tr>
<th>Determined coefficients</th>
<th>Extracted parameters (in GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 = 6.96198, p_1 = 0.49350 )</td>
<td>( \Delta \varepsilon_{21} = 14.033, f_{s1} = 183.36 )</td>
</tr>
<tr>
<td>( A_2 = 6.82613, p_2 = 0.62870 )</td>
<td>( \Delta \varepsilon_{21} = 18.550, f_{s1} = 120.50 )</td>
</tr>
<tr>
<td>( A_3 = 2.88101, p_3 = 0.30103 )</td>
<td>( \Delta \varepsilon_{21} = 4.3738, f_{s1} = 311.69 )</td>
</tr>
<tr>
<td>( A_4 = -1.59396, p_4 = 0.75867 )</td>
<td>( \Delta \varepsilon_{21} = 6.6259, f_{s1} = 71.707 )</td>
</tr>
<tr>
<td>( A_5 = 1.25497, p_5 = 0.10192 )</td>
<td>( \Delta \varepsilon_{21} = 1.7124, f_{s1} = 592.87 )</td>
</tr>
<tr>
<td>( A_6 = 0.29796, p_6 = 0.87280 )</td>
<td>( \Delta \varepsilon_{21} = 2.2011, f_{s1} = 35.322 )</td>
</tr>
<tr>
<td>( A_7 = 0.04555, p_7 = 0.94877 )</td>
<td>( \Delta \varepsilon_{21} = 0.89826, f_{s1} = 13.654 )</td>
</tr>
<tr>
<td>( A_8 = 0.00415, p_8 = 0.98790 )</td>
<td>( \Delta \varepsilon_{21} = 0.34251, f_{s1} = 3.1616 )</td>
</tr>
</tbody>
</table>

Fig. 2 Debye parameters retrieved from Cole-Cole models of muscle.

Fig. 3 Reflection coefficients obtained from theory and numerically derived using the extracted Debye parameters for each Cole–Cole term.
planewave with polarization in the $z$-direction impinges into the sphere from the $-x$-direction. The amplitude of the incident electric field is 1 V/m. The waveform of the incident electric field is the Gaussian pulse with $T_0 = 0.385$ ns and $\alpha_0 = 0.146$ ns. The spatial resolution and time step for the FDTD calculation are 2 mm and $\Delta t = 3.85$ ps, respectively. The total number of cells in analysis model is $141 \times 141 \times 141$. Eight-layered PMLs are placed outside the analysis region to absorb the outgoing EM waves. The number of calculation time steps is 5000. All numerical analyses are performed on a workstation (Intel Xeon E5-2697v3 @ 2.6 GHz, 512-GB memory) with 22 parallel threads. The calculation time is approximately 320 s.

Figs. 6 (a) and (b) show the time-domain electric field calculated at two observation locations ($x, z$) = ($-0.08$ m, 0 m) and ($-0.04$ m, 0 m) with respect to the center of the sphere, respectively, as shown in Fig. 5. The Mie solutions are calculated by the inverse Fourier transform of those in the frequency domain [45]. Note that the Mie solutions cannot be obtained at the center of the sphere due to the divergence of the Hankel function with zero argument. As seen from Figs. 6 (a) and (b), the time-domain electric fields derived by the proposed FDTD method agree well with the analytical Mie solutions in the time domain, demonstrating the validity of our proposed method. They also exactly match the numerical simulation results, shown as the dash-dotted blue lines, obtained by the FDTD method described in [31]. The amplitude of the pulse decreases as it advances into the dielectric sphere. From Fig 6 (b), we observe small differences in the late-time response at around 4.8 ns. The differences are attributed from errors in modeling the sphere using the staggered grid of the FDTD method. The EM wave travelling along the curved sphere surface will take a longer time to reach the back of the sphere in numerical simulations since the staircase approximation of the sphere boundary results in the increased travel distance of EM pulses. The effect seems to be smaller for the location ($x, z$) = ($-0.08$ m, 0 m) in Fig. 6 (a), since most of the diffracted EM wave energy penetrating the sphere from the back is absorbed and cannot reach the front of the sphere where the observation point is located. It is also observed that the pulse penetrating the sphere is oscillatory and has both positive and negative values even though the incident electric field has only positive values.

Fig. 7 (a) and (b) shows the electric field distribution inside the homogeneous muscle sphere characterized by four Cole–Cole terms at 500 MHz using the on-the-fly Fourier transform with a single FDTD run and that obtained from Mie theory. A strong electric field appears not only close to the surface but also near the center of the sphere with the peak shifted to an anterior location due to the concentration...
of the EM wave penetrating the sphere. The increased absorption is due to a combination of high dielectric constant and the curvature of the sphere. The result shows a similar trend to those in the literature [46]. Although not shown here, it is noteworthy that the electric field distributions, obtained using the extracted parameters, at other frequencies in the valid range between 10 MHz and 1 GHz are also in good agreement with the analytical Mie results.

Fig. 8 shows $S_A$, $S_A L$, $W_0$, and $W_L$ with respect to time at $(x, z) = (-0.04 \text{ m}, 0 \text{ m})$, where $W_0$ and $W_L$, calculated using
\[
W_0 = \frac{\Delta t}{\rho} \rho, W_L = \frac{\Delta t}{\rho} \rho
\]  
are the transient specific energy absorption and transient specific energy loss, respectively. They represent the energy that is stored or dissipated into the medium over a single time step. Incident power density of the Gaussian pulse with a full width at half maximum (FWHM) of approximately 0.172 ns is also indicated as a grey solid line in Fig. 8. As shown in the figure, $W_0$ shows greater variation than $W_L$, which is the energy loss by dissipation into the medium. It is clearly observed that $W_0$ exhibits a negative value at around 2 ns due to the excessive release of stored energy while $W_L$ only takes positive values. It is seen that FWHM of $W_L$ is approximately 0.280 ns, wider than that of incident power density due to dispersive nature of Cole–Cole model. As might be expected, while $S_A$ may decrease due to the release of stored energy, $S_A L$ monotonically increases to a specific value, which indicates the total amount of specific energy loss within a single cycle of the EM pulse. At the end of the pulse illumination, both $S_A$ and $S_A L$ reach the same value of the specific energy absorption after the EM pulse diminishes to zero or after 5 ns as shown in Fig. 8. $S_A L$ is calculated using the EM pulse amplitude decreases to a negligible level at the 10000th time step. The figure shows that the highest peak of $S_A$ appears at the surface of the sphere. There are two other peaks, which appear near but not exactly at the center of the sphere. The shift in the location of the SA peaks is attributed to the transmission properties caused by high dielectric constants of the medium and the refraction of the incident plane wave propagating along the curved surface. The maximum value of $S_A$ is $1.062 \times 10^{-14}$ J/kg, almost the same as $1.038 \times 10^{-14}$ J/kg, the $S_A$ maximum derived in [31]. The difference in the maximum $S_A$ between the two methods is approximately 2.26%. The difference is caused by the different formulation of the Debye terms as mentioned in Subsection 3.1.

To further confirm the correctness of the proposed algorithm for determining the specific energy absorption and specific energy loss, we apply a sinusoidal waveform oscillating at 500 MHz as an incident plane wave to the same sphere model. Fig. 10 shows the electric field $E_z$ and the specific energy absorptions $S_A$, $S_A L$, $W_0$, and $W_L$ at the observation point $(x, z) = (-0.04 \text{ m}, 0 \text{ m})$. It is interesting that both $W_0$ and $W_L$ oscillate at a higher frequency of 1 GHz, twice that of the electric field at the same point. It can also be observed in the figure that $W_0$ may exhibit a negative value, indicating the excessive release of the stored energy. Meanwhile, $W_L$ shows only positive values. To compare these results with analytical solutions, we calculate the SAR over a period of the analysis frequency using the following equations derived from (33) and (37):
\[
SAR(k) = \frac{1}{\rho W_0} \int_{kT_0}^{(k+1)T_0} p_L(t) dt = \frac{n}{T_0} \sum_{i=kT_0/\Delta t}^{(k+1)T_0/\Delta t} W_L^i
\]  
where $kT_0 \leq i\Delta t \leq (k+1)T_0$, $T_0$ is the time period, and $k$ is the index number of the period. Fig. 11 shows the specific absorption rates calculated using $W_0$ or $W_L$ in (38) at the observation points $(x, z) = (-0.08 \text{ m}, 0 \text{ m})$ and $(0.04 \text{ m}, 0 \text{ m})$. The theoretical values are determined from
\[
SAR^{\text{Mie}} = \frac{\sigma |E|^{\text{Mie}}_z^2}{2\rho}
\]  
where $E^{\text{Mie}}_z$ is the electric field strength obtained from Mie theory. Specific absorption rates SAR$^{\text{Mie}}$ are also indicated in Fig. 11 as the dashed lines. As can be seen from Fig. 11, SARs calculated using either $W_0$ or $W_L$ closely matched with the Mie solutions when the EM waves reach their steady state after five periods, demonstrating the correctness of the expressions for calculating the SAs in (22) and (31). Therefore, the average values of $W_0$ and $W_L$ over a period are the same. $W_0$ and $W_L$ at times of less than five periods are in a transient stage, resulting in differences from the
theory. It should be emphasized that for the sinusoidal sources, the average values of both $W_0$ and $W_L$ match each other in the steady state but they will be different for a pulse waveform, where the steady state is not sustained. From the physical meanings of $W_0$ and $W_L$, it is worth mentioning that $W_L$ should be the only quantity used as a heat source when calculating the temperature elevation due to the exposure of EM pulses.

4. EM Pulse Exposure to Human Head

4.1 Human Head Models and Analysis Parameters

To calculate the EM energy deposition into a human head, numerical human head models are extracted from the Japanese adult male and female models TARO and HANAKO, respectively, which were developed by the National Institute of Information and Communications Technology, Tokyo, Japan [47]. Fig. 12 illustrates the head models, which comprise of 26 different tissues and organs with a resolution of 2 mm. The dielectric properties of biological tissues are represented by the four-term Cole–Cole model and their Debye parameters are extracted using the proposed parameter extraction method and used in the FDTD simulations. An EM plane wave with a polarization direction in the $z$-axis and a magnitude of 1 V/m is used to impinge on the head models from the front as shown in Fig. 12. The incident pulse used is the Gaussian waveform expressed by (35) with the parameter values of $T_0 = 0.385$ ns and $\alpha_0 = 0.146$ ns. The time step interval is 3.84 ps and the total size of the analysis models with eight-layer PMLs is $166 \times 156 \times 172$ cells for both models. The total computation time for TARO and HANAKO models are 759 s (12 min 39 s) and 750 s (12 min 30 s) for 10,000 time steps, respectively.

4.2 Energy Deposition of EM Planewave into Human Head

The specific energy loss is calculated at each time step using the extracted parameters of the Cole–Cole models. Fig. 13 (a) – (e) illustrate the calculated transient specific energy loss using (32) and (37) at 100, 150, 200, 250, and 300 steps, respectively. In Fig. 13 (a), the EM pulse first reaches the human nose from the front and, hence, the energy is absorbed instantaneously. At 150 steps (Fig. 13 (b)), the pulse illuminates the whole face and begins to penetrate the head, resulting in high energy absorption between the eyes, around the eyes, and at the tip of the nose. At 200 steps (Fig. 13 (c), after the EM pulse precursor reaches the top of the head, the energy absorption of the EM pulse occurs at deeper tissues, and the pulse energy begins to concentrate along the $z$-axis since the curvature of the head model is larger in the azimuth plane than that in the elevation plane. It is observed...
that energy absorption in the eyes occurs from this step, with a delay compared with that of other superficial tissues such as skin and fat. After 250 steps, a strip of specific energy loss is observed in the human frontal brain as shown in Fig. 13 (d). When the EM pulse precursor completely passes through the human head at around 300 steps, the energy continues to be absorbed at the deeper biological tissues inside the head. As shown in Fig. 13 (e), the pulse energy absorption at the deeper tissues almost remains in a strip along the $z$-axis, and it does not spread across the entire head. The energy absorption in the head is considered negligible after approximately 2000 time steps for the Gaussian pulse used in this study, and the simulations are stopped. By using the proposed extracted parameters, for the first time, we can derive the transient specific energy loss at every time steps, which cannot be obtained using the conventional method. Note again that the specific energy loss corresponds to the temperature rise in biological tissues and has physical meanings, while the specific energy absorption includes the stored energy, which may be released over time. Fig. 14 indicates the specific energy absorption for the TARO and HANAKO head models. The result for TARO is also consistent with that in [31], demonstrating the validity of the proposed method. As indicated in Fig. 14, $SA$ shows higher values at the skin and the tissues just beneath the skin such as muscle. It also shows moderate values in the deep tissues such as the cerebrospinal fluid (CSF) at locations $p_1$ and $p_2$, the tongue, and cartilage. $SA$ in HANAKO tends to be higher due to the smaller volume of the head with the larger curvature and the larger portion of CSF than that in TARO. The maximum values of $SA$ in TARO and HANAKO are $5.58 \times 10^{-14}$ and $4.78 \times 10^{-14}$ J/kg, respectively. Note that the results of $SA$ obtained here can be used in compliance with international exposure standards [48, 49].

Figs. 15 (a) and (b) show the specific energy loss and its transient component at two observation points indicated in Fig. 14 as $p_1$ (CSF) and $p_2$ (CSF) for TARO and HANAKO, respectively. It is observed that most of the pulse energy is rapidly absorbed into the tissue due to the first incoming pulse which penetrates the head from the front for both $p_1$ and $p_2$ in TARO. However, for HANAKO, the same trend only applies at $p_2$, which is located near the head surface. At the observation point $p_1$, we observe almost the same energy of the second wave. This is due to the diffraction of the EM pulse around the head. The larger curvature and smaller size of HANAKO model also contribute to higher absorption inside the deep tissues. It should be emphasized that it is very important to take into account the dispersion characteristic in order to quantify correctly amount of absorbed energy or electric/magnetic field strengths inside biological tissues since they are directly related to many biological effects such as body temperature elevation and nervous stimulus. Inaccurate dosimetric results may lead to misinterpretation of biological effects observed in experiments. In conclusion, it has been demonstrated that the numerical method proposed in this paper can fully take into account the frequency dependence of the dielectric properties of biological media, provides accurate dosimetric results, and gives physical insight into the interaction.

**Fig. 12** Numerical human head models TARO and HANAKO used in simulations of EM pulse exposures.

**Fig. 13** Distribution of the transient specific energy loss in a human head model at (a) 100, (b) 150, (c) 200, (d) 250, and (e) 300 time steps, and (f) $xz$-plane distribution of the transient energy loss at 300 time steps.
In this paper, we have proposed a Debye parameter extraction method for the Cole–Cole model, which is used to represent the dielectric properties of biological media. We have also developed a frequency-dependent FDTD method incorporating the parameters extracted via the formulation of the update electric field using the ADE method and we also provided an algorithm for calculating specific energy loss and its transient component, which cannot be determined using the conventional approach. The validity of the parameter extraction method has been demonstrated via comparison between numerical results and those obtained by an analytical method. It was shown that the proposed FDTD scheme is more accurate at higher frequencies owing its different formulation, which provides second-order accuracy in both time and space. Numerical simulations of a homogeneous muscle sphere illuminated by an EM pulse were carried out. The field distributions inside the sphere were in good agreement with those obtained from Mie theory, both in the time-domain response and in the field distribution at a frequency of 500 MHz. The correctness of the algorithm for calculating energy absorbed into biological media was confirmed. In addition, physical insights into specific energy absorption, which is conventionally used as a metric, and specific energy loss were provided. Further studies are to investigate the temperature elevation due to EM pulse exposure, a compliance study on various human subjects, and the extension of the proposed method to various EM applications such as ground penetrating radar and medical imaging.

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References


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