INVITED PAPER

Optimization by Neural Networks in the Coherent Ising Machine and its Application to Wireless Communication Systems

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SUMMARY  Recently, new optimization machines based on non-silicon physical systems, such as quantum annealing machines, have been developed, and their commercialization has been started. These machines solve the problems by searching the state of the Ising spins, which minimizes the Ising Hamiltonian. Such a property of minimization of the Ising Hamiltonian can be applied to various combinatorial optimization problems. In this paper, we introduce the Coherent Ising machine (CIM), which can solve the problems in a milli-second order, and has higher performance than the quantum annealing machines especially on the problems with dense mutual connections in the corresponding Ising model. We explain how a target problem can be formulated as Eq. (1) [6].

where $\sigma_i$ is the state of the $i$th Ising spin, $\sigma_i \in \{-1, 1\}$, $J_{ij}$ is the mutual interaction between the $i$th and the $j$th Ising spins, $h_i$ is the external magnetic field of the $i$th Ising spin, and $N$ is the number of the Ising spins, respectively. There are various combinatorial optimization problems, which can be formulated as Eq. (1) [6].

1. Introduction

New high-speed optimization machines based on non-silicon physical systems, such as quantum annealing machines, e.g. [1]-[5], have been developed and their commercialization has been started. These machines search the state of the Ising spins $\sigma$ corresponding to the minimum of the Ising Hamiltonian $H(\sigma)$,

$$H(\sigma) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} J_{ij} \sigma_i \sigma_j + \sum_{i=1}^{N} h_i \sigma_i$$

where $\sigma_i$ is the state of the $i$th Ising spin, $\sigma_i \in \{-1, 1\}$, $J_{ij}$ is the mutual interaction between the $i$th and the $N$th Ising spins. Therefore, $H(\sigma)$ becomes possible to obtain their solutions in a very short time by the quantum annealing machines.

On the other hand, the Coherent Ising Machine (CIM) [2]-[5] can deal with the fully connected mutual interactions of $J_{ij}$ among $N$ Ising spins. Therefore, the CIM can be applied to larger problems than the D-wave. Ref. [8] has shown that the CIM has better performance than the D-wave especially for the large problems with dense connections of $J_{ij}$. In the previous researches in Refs. [4], [5], it has been shown that the CIM can obtain the solution of a benchmark Max-Cut problem, in several hundreds of micro-seconds, by the implemented real machine experiments.

Such a high-speed combinatorial optimization will be effective for various communication systems. It becomes possible to optimize the communication systems in real-time, even for dynamically changing environments of various factors, such as communication traffic, channel states and so on. By the network virtualization technologies, various factors and parameters of the communication systems become tunable. Such tunable parameters can be optimized by the high-speed optimization machines described above, for keeping high

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performance for those systems.

In this paper, we introduce the CIM for the high-speed combinatorial optimization. We explain a method how to solve the combinatorial optimization problems by the CIM, based on the conventional mutually connected neural networks. As an example of benchmark combinatorial optimization problems, we apply the CIM to the TSP. We show several experimental results of the real machine of the CIM. We also apply the CIM to optimization problems in a wireless communication system, with calculating $J_{ij}$ and $h_i$ of the Ising model to solve those problems.

2. Coherent Ising Machines

The first CIM was proposed based on an injection locked laser system, which is shown in Fig. 1. By the mutual interactions among the slave lasers, $J_{ij}$, oscillation mode of each slave laser will converge to orthogonal polarization states, right or left circular polarization. Those states of the polarization are used as the state of the Ising spins, $\sigma_i$. Therefore, this system minimize the Ising Hamiltonian, $H(\sigma)$, and can solve combinatorial optimization problems. However, in order to apply this CIM to large-scale problems, it is necessary to increase the number of the slave lasers corresponding to Ising spins, but it is difficult to prepare a large number of mutual connections for the laser network based CIM.

For large-scale problems, the delayed-feedback type CIM [3] and the measurement-and-feedback type CIM [4], [5] have been proposed. In these machines, optical pulses are used as the Ising spins $\sigma_i$. The optical pulses are running on a long optical fiber, which has about 1 kilometer length or longer. Figure 2 shows a structure of the measurement-and-feedback type CIM. The mutual interactions among the Ising spins are calculated at the FPGA module. Therefore, the mutual interactions are programmable by a software and even full connection interactions can be realized. In Refs. [4], [5], high computation speed and high performance of the CIM are shown with experimental results.

The CIM searches the state of Ising spins, $\sigma$, corresponding to the minimum of $H(\sigma)$ in Eq. (1). Therefore, the CIM can solve the combinatorial optimization problems, which can be formulated as Eq. (1). In the most of previous researches on the CIM, Max-Cut problems are selected as a benchmark problem [4], [5]. The objective function of the Max-Cut problem is almost the same as the Eq. (1), and it is easy to apply the CIM.

Other various combinatorial optimization problems can also be formulated as Eq. (1) [6]. In Ref. [7], the mutually connected neural network to solve the TSP has been proposed. The mutually connected neural network decreases the following energy function,

$$E(x) = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} w_{i,j} x_i x_j + \sum_{i=1}^{N} \theta_i x_i,$$

where $x_i$ is the state of neurons, $x_i \in \{0, 1\}$, $w_{i,j}$ is the connection weight between the $i$th and the $j$th neurons, $\theta_i$ is the threshold of the $i$th neuron, respectively. The optimization algorithms using the mutually connected neural networks solve the problems by minimizing dynamics of the energy function. The mutually connected neural networks have been applied to various combinatorial optimization problems [7], [9], [10]. Although there is a difference in the output state of the Ising spins in CIM $\{-1, 1\}$ and the neurons $\{0, 1\}$, by transforming the outputs, the CIM can solve the problems, which the mutually connected neural networks have been applied to.

To solve a combinatorial optimization problem by
the CIM, the mutual interactions between the Ising spins, \( J_{i,j} \), and the external magnetic field, \( h_i \), have to be appropriately set to minimize the target objective function. In order to obtain \( J_{i,j} \) and \( h_i \), first we have to define the state of the Ising spins, \( \sigma \), to be appropriately set to minimize the target objective function. By comparing the formulated function of \( \sigma \) and Eq. (1), we can derive \( J_{i,j} \) and \( h_i \). By using the derived \( J_{i,j} \) and \( h_i \), the CIM solves the target problem in quite high speed and high performance.

3. Applying Coherent Ising Machines to Traveling Salesman Problems

3.1 Mutually Connected Neural Networks and Energy Function

The state of the neurons in the mutually connected neural network can be defined by the following equation,

\[
x_i(t + 1) = u \left[ \sum_{j=1}^{N} w_{i,j} x_j - \theta_i \right],
\]

where, \( u \) is the Heaviside step function, \( u[y] = 1 \) for \( y > 0 \) and \( u[y] = 0 \) for \( y < 0 \). When the mutual connections are symmetric, \( w_{i,j} = w_{j,i} \), the self-feedback connections are 0, \( w_{i,i} = 0 \), for all \( i \) and \( j \), and the state of neurons are updated asynchronously, the energy function in Eq. (2) always decreases. The state of the mutually connected neural network will converge to minimum of the Eq. (2) by iterative updates using Eq. (3). This minimization dynamics has been applied to various combinatorial optimization problems to search their solutions.

To apply the minimization of the energy function to a combinatorial optimization problem, the state of the target optimization problem has to be defined by the state of the neurons, \( x_i \), at first. Then, the objective function of the target problem is formulated as the function of the neurons’ states, \( x_i \). By comparing the formulated objective function with the energy function in Eq. (2), the connection weights \( w_{i,j} \) and the threshold \( \theta_i \) can be derived. By iteratively updating the neural network using Eq. (3) with the derived \( w_{i,j} \) and \( \theta_i \), its state converges to \( x \) corresponding to the minimum of \( E(x) \) and the state of the neural network at this state shows the solution of the target problem.

3.2 Mutually Connected Neural Network to solve TSP

In Ref. [7], the mutually connected neural network was applied to the TSP, given the position of \( n \) cities, find the minimum length tour visiting each city exactly once. For solving the TSP by the mutually connected neural network, the tour has to be expressed by the state of neurons. In the method of Ref. [7], \( n \times n \) neurons, \( x_{ij} \), are prepared for the \( n \) city-problem, where the index \( i \) corresponds to the city label and the index \( j \) corresponds to the visiting order, respectively. \( x_{ij} = 1 \) means the city \( i \) is visited at the \( j \)th order. This neural network is updated by the following equation,

\[
x_{ij}(t + 1) = u \left[ \sum_{k=1}^{n} \sum_{l=1}^{n} w_{ij,kl} x_{kl} - \theta_{ij} \right],
\]

and the energy function of this neural network becomes as follows,

\[
E(x) = -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} w_{ij,kl} x_{ij} x_{kl} + \sum_{i=1}^{n} \sum_{j=1}^{n} \theta_{ij} x_{ij},
\]

where, \( w_{ij,kl} \) is the connection weight between the \( ij \)th and the \( kl \)th neurons, \( \theta_{ij} \) is the threshold for the \( ij \)th neuron, respectively.

Based on the above definition of the neurons, the tour length of the TSP can be formulated as follows,

\[
E^{TSP1}(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{i,k} x_{ij} (x_{kj+1} + x_{kj-1}),
\]

where, \( d_{i,k} \) is the distance between the cities \( i \) and \( k \). In order to obtain a feasible solution of the TSP, visiting each city exactly once, we also need to minimize the following constrain terms. For making the solution visiting each city only once, the following constraint, \( E^{TSP2} \), has to be minimized to 0,

\[
E^{TSP2}(x) = \sum_{i=1}^{n} \left( \sum_{j=1}^{n} x_{ij} - 1 \right)^2.
\]

Also, because it is not possible to visit more than one city at the same time, the following constraint, \( E^{TSP3} \), has to be 0,

\[
E^{TSP3}(x) = \sum_{j=1}^{n} \left( \sum_{i=1}^{n} x_{ij} - 1 \right)^2.
\]

By transforming \( E^{TSP1} \), \( E^{TSP2} \) and \( E^{TSP3} \) to the form of the energy function in Eq. (5), the connection weights \( w_{ij,kl} \) and the threshold \( \theta_{ij} \) to minimize each objective function can be derived. In the following calculations, it should be noted that the self-feedback connection should be 0, \( w_{i,i,j} = 0 \), therefore the coefficients on \( x_{ij} x_{ij} \) should be avoided by using \( x_{ij} x_{ij} = x_{ij} \). \( E^{TSP1} \) can be transformed to the following equation,

\[
E^{TSP1}(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} d_{i,k} (\delta_{j+1,l} + \delta_{j-1,l}) x_{ij} x_{kl},
\]
where, $\delta_{i,k}$ is the Kronecker delta, $\delta_{i,k} = 1$ when $i = k$, $\delta_{i,k} = 0$ otherwise. In this transformation, $x_{kj+1}$ and $x_{kj-1}$ in Eq.(6) are replaced by $x_{kl}$, with introducing the Kronecker delta, $\delta_{j+1,l}$ and $\delta_{j-1,l}$ for each term, respectively. By comparing the coefficients of $x_{ij}x_{kl}$ in Eqs. (9) and (5), the connection weights $w_{ij,kl}^{TSP1}$ to minimize $E_{TSP1}^{T}(x)$ can be derived as follows,

$$w_{ij,kl}^{TSP1} = -2d_{ij}k(\delta_{j+1,i} + \delta_{j-1,i}).$$  \hspace{1cm} (10)

Equation (7) can be transformed as follows, with avoiding self-feedback connections and using $x_{ij}x_{ij} = x_{ij}$,

$$E_{TSP2}^{T}(x) = \sum_{i=1}^{n} \left( \sum_{j=1}^{n} \sum_{l=1}^{n} x_{ij}x_{il} - 2 \sum_{j=1}^{n} x_{ij} + 1 \right)$$

$$= \sum_{i=1}^{n} \left( \sum_{j=1}^{n} \sum_{l=1}^{n} (1 - \delta_{j,l})x_{ij}x_{il} + \sum_{j=1}^{n} \sum_{l=1}^{n} \delta_{j,l}x_{ij}x_{il} - 2 \sum_{j=1}^{n} x_{ij} + 1 \right)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} \delta_{i,k}(1 - \delta_{j,l})x_{ij}x_{kl} - \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} + n.$$  \hspace{1cm} (11)

By comparing the coefficients of $x_{ij}x_{kl}$ and $x_{ij}$ in Eqs.(11) and (5), the connection weights $w_{ij,kl}^{TSP2}$ and the thresholds $\theta_{ij}^{TSP2}$ to minimize $E_{TSP2}^{T}(x)$ are obtained as follows,

$$w_{ij,kl}^{TSP2} = -2\delta_{i,k}(1 - \delta_{j,l}),$$

$$\theta_{ij}^{TSP2} = -1.$$  \hspace{1cm} (13)

In a similar way to the derivation of Eq.(11), Eq. (8) can be transformed to the function of the neuron states $x_{ij}$, and the connection weights $w_{ij,kl}^{TSP3}$ and the thresholds $\theta_{ij}^{TSP3}$ to minimize $E_{TSP3}^{T}(x)$ are obtained as follows,

$$w_{ij,kl}^{TSP3} = -2\delta_{j,l}(1 - \delta_{i,k}),$$

$$\theta_{ij}^{TSP3} = -1.$$  \hspace{1cm} (15)

In order to solve the TSP, $E_{TSP1}^{T}$, $E_{TSP2}^{T}$ and $E_{TSP3}^{T}$ have to be minimized at the same time, so the overall objective function is defined as follows,

$$E_{TSP}^{T} = AE_{TSP1}^{T} + BE_{TSP2}^{T} + CE_{TSP3}^{T},$$  \hspace{1cm} (16)

where $A$, $B$ and $C$ are the weight parameters for each term. The connection weights $w_{ij,kl}^{T}$ and the threshold $\theta_{ij}^{T}$ to minimize $E_{TSP}^{T}$ for solving the TSP are obtained as follows, using $w_{ij,kl}^{TSP1}$, $w_{ij,kl}^{TSP2}$, $w_{ij,kl}^{TSP3}$, $\theta_{ij}^{TSP2}$ and $\theta_{ij}^{TSP3}$ derived in Eqs. (10),(12)-(15),

$$w_{ij,kl}^{T} = -Ad_{ij}(\delta_{j+1,i} + \delta_{j-1,i}) - B\delta_{i,k}(1 - \delta_{j,l}) - C\delta_{j,l}(1 - \delta_{i,k}),$$

$$\theta_{ij}^{T} = -(B + C)/2.$$  \hspace{1cm} (18)

By iterating the neural network using Eq. (4) with $w_{ij,kl}^{T}$ and $\theta_{ij}^{T}$, its state will converge to a solution of the TSP.

However, the performance of this neural network is poor, because the state easily gets stuck at a local minimum. Since the neural network has only-decreasing property, the search easily stops at the local minimum of the energy function. Therefore, in the previous researches, stochastic or chaotic fluctuations are used to improve the performance of the neural network for combinatorial optimization [11]-[14].

### 3.3 Coherent Ising Machines to solve the TSP

In order to apply the CIM to the TSP explained in the previous subsection, we calculate the CIM’s mutual interaction $J_{ij,kl}$ and the external magnetic field $h_{ij}$ based on the connection weights $w_{ij,kl}^{T}$ and threshold $\theta_{ij}^{T}$. There is a difference in the output states of the neural network and the CIM. The state of each neuron in the neural networks $x_{ij}$ takes 0 or 1 values, $x_{ij} \in \{0,1\}$. On the other hand, the Ising spins of the CIM $\sigma_{ij}$ takes $-1$ or 1 values, $\sigma_{ij} \in \{-1,1\}$. Such a difference in the output states can be easily resolved by defining a neural network having $-1$ or 1 outputs as follows,

$$\tilde{x}_{ij}(t + 1) = r \left[ \sum_{k=1}^{n} \sum_{l=1}^{n} \tilde{w}_{ij,kl} \tilde{x}_{kl} - \tilde{\theta}_{ij} \right],$$  \hspace{1cm} (19)

where $r[y] = 1$ for $y > 1$, $r[y] = -1$ for $y < 0$, $\tilde{w}_{ij,kl}$ and $\tilde{\theta}_{ij}$ are the connection weights and the thresholds of the neural networks with 1 or $-1$ outputs, respectively. By using $\tilde{x}_{ij} = 2x_{ij} - 1$, the internal state of Eq. (4) can be reformulated as follows,

$$\sum_{k=1}^{n} \sum_{l=1}^{n} w_{ij,kl}x_{kl} - \theta_{ij} = \sum_{k=1}^{n} \sum_{l=1}^{n} \frac{w_{ij,kl}x_{kl}}{2} - \left( \theta_{ij} - \frac{\sum_{k=1}^{n} \sum_{l=1}^{n} w_{ij,kl}}{2} \right).$$  \hspace{1cm} (20)

By comparing Eqs.(19) and (20), we can obtain $\tilde{w}_{ij,kl}$ and $\tilde{\theta}_{ij}$. If we use the $-1$ or 1 output neuron, $\tilde{x}_{ij}(t)$, to update the neural network, we have to set $\tilde{w}_{ij,kl}$ and $\tilde{\theta}_{ij}$ as follows,

$$\tilde{w}_{ij,kl} = \frac{w_{ij,kl}}{2} = J_{ij,kl},$$  \hspace{1cm} (21)
\[ \tilde{\theta}_{ij} = \theta_{ij} - \sum_{k=1}^{n} \sum_{l=1}^{n} \frac{w_{ijkl}}{2} = h_{ij}. \]  

By obtaining \( J_{ij,kl} \) and \( h_{ij} \) by Eqs. (21) and (22), the CIM’s high-speed optimization performance becomes applicable to various combinatorial optimization problems where the mutually connected neural networks can be applied.

3.4 Experiments on Coherent Ising Machines

In order to solve the TSP by the real machine of the CIM, we obtain \( J_{ij,kl}^{\text{TSP}} \) and \( h_{ij}^{\text{TSP}} \) by Eqs. (21) and (22) with \( w_{ij,kl}^{\text{TSP}} \) and \( \theta_{ij}^{\text{TSP}} \). We have programmed the obtained \( J_{ij,kl}^{\text{TSP}} \) and \( h_{ij}^{\text{TSP}} \) on the FPGA of the real machine of the CIM shown in Fig. 2.

Figure 3 shows the time series of the Ising spins of the CIM solving a TSP. The amplitude of the pump pulse is gradually increased [15], and each state of the spins converges to plus or minus values within 5ms. The in-phase components of the spins show the obtained solution. Table 1 shows the results on 5 different TSPs, city10a[7], city10b, city10c[13], gr17[16], city20[14]. The performances are evaluated by the gap between the average obtained solutions and the already known optimum solutions. Although these are small test problems, the experimental results show that the CIM obtains the solutions with 0.000% gap from the optimum that means the optimum solution can be obtained in every run.

![Fig. 3](image)

Fig. 3  Time series of the Ising spins of the CIM solving a TSP.

4. Application of CIM to Wireless Communication Systems

In wireless communication systems, there are various combinatorial optimization problems, such as channel assignment, power allocation, scheduling and so on. The CIM can also solve those problems in a quite high-speed. In this section, we apply the CIM to typical examples of optimization problems in wireless communication systems.

4.1 Application to a Channel Assignment Problem

When many wireless access points exist in a same area, their communications interfere each other. In such an environment, it is possible to minimize the interference by appropriately assigning the channels to each access point. In this subsection, we formulate the CIM to solve the channel assignment problem for maximizing the signal to interference ratio (SIR), which corresponds to the channel capacity. We assume a wireless network environment, in which many wireless devices are communicating through each corresponding access point.

To define the capacity maximization, the total of the received power strengths of the desired signals and the interferences at the access point \( i \) are defined as follows,

\[
S_i = \sum_{u \in U_i} P d_{iu}^{-\alpha},
\]

\[
I_i = \sum_{k(k \neq i), CH_i = CH_k} \sum_{v \in U_k} P d_{iv}^{-\alpha},
\]

where, \( P \) is the transmission power, \( U_i \) is the users communicating through the access point \( i \), \( CH_i \) is the channel assigned for the access point \( i \), \( d_{iu} \) is the distance between the access point \( i \) and the mobile user terminal \( u \), and \( \alpha \) is the path loss exponent, respectively. For maximizing \( S_i/I_i \) for all access points, we formulate a neural network to minimize \( I_i/S_i \). To obtain the symmetric connections for the neural network, the objective function is defined as follows,

\[
E_{\text{OBJ}}^{\text{WLAN}} = \sum_i \frac{1}{k(k \neq i, CH_i = CH_k)} \left( \frac{\sum_{v \in U_k} d_{iv}^{-\alpha}}{\sum_{v \in U_k} d_{iv}^{-\alpha}} + \frac{\sum_{u \in U_i} d_{iu}^{-\alpha}}{\sum_{u \in U_i} d_{iu}^{-\alpha}} \right). \tag{25}
\]

To solve the channel assignment problem by the CIM, we formulate a mutually connected neural network with \( x_{ij} \in \{0, 1\} \) to minimize the formulated objective function in Eq. (25), at first. The state of the \( x_{ij} \) is defined as follows, when \( x_{ij} = 1 \), the channel \( j \) is

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Table 1  Results of the CIM applied to TSPs.

<table>
<thead>
<tr>
<th>Problem Size</th>
<th>Average gap from optimum(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>City10a 10</td>
<td>0.000</td>
</tr>
<tr>
<td>City10b 10</td>
<td>0.000</td>
</tr>
<tr>
<td>City10c 10</td>
<td>0.000</td>
</tr>
<tr>
<td>gr17 17</td>
<td>0.124</td>
</tr>
<tr>
<td>City20 20</td>
<td>0.043</td>
</tr>
</tbody>
</table>
assigned to the access point $i$. By using $x_{ij}$, Eq. (25) can be formulated as follows,

$$E_1^{\text{WLAN}}(x) = \sum_{i=1}^{N_{\text{AP}}} \sum_{j=1}^{N_{\text{CH}}} \sum_{k=1}^{N_{\text{CH}}} \sum_{l=1}^{N_{\text{CH}}} \delta_{j,l}(1 - \delta_{i,k}) \left( \sum_{v \in U_i} d_{iv}^{-\alpha} \delta_{v,u} \right) \left( \sum_{k \in U_i} d_{kv}^{-\alpha} \delta_{k,u} \right) x_{ij} x_{kl},$$

(26)

where, $N_{\text{AP}}$ is the number of access points and $N_{\text{CH}}$ is the number of available channels, respectively. In this problem formulation, we also assume that each access point can use only one channel for each. Then, the following constraint term is also required to be minimized,

$$E_2^{\text{WLAN}}(x) = \sum_{i=1}^{N_{\text{AP}}} \left( \sum_{j=1}^{N_{\text{CH}}} x_{ij} - 1 \right)^2.$$

(27)

To get the solution of the defined channel assignment problem, we minimize $E^{\text{WLAN}}(x) = A E_1^{\text{WLAN}}(x) + B E_2^{\text{WLAN}}(x)$, where $A$ and $B$ are weight parameters for each objective energy function. By transforming $E^{\text{WLAN}}(x)$ to the form of the neural network energy function in Eq. (5), the connection weights $w_{ij,kl}^{\text{WLAN}}$ and the threshold $\theta_{ij}^{\text{WLAN}}$ can be derived as follows,

$$w_{ij,kl}^{\text{WLAN}} = A \delta_{j,l}(1 - \delta_{i,k}) \left( \sum_{v \in U_i} d_{iv}^{-\alpha} \delta_{v,u} \right) \left( \sum_{k \in U_i} d_{kv}^{-\alpha} \delta_{k,u} \right) - B \delta_{i,k}(1 - \delta_{j,l}),$$

(28)

$$\theta_{ij}^{\text{WLAN}} = -\frac{B}{2}.$$

(29)

$J_{ij,kl}$ and $h_{ij}$ for solving this problem by the CIM can be obtained by using Eqs. (21) and (22). By using the obtained $J_{ij,kl}$ and $h_{ij}$, the CIM will converge to the state corresponding to the solution of the channel assignment problem, in a quite high-speed.

### 4.2 Optimization of Transmission Power

In the previous subsection, the transmission power $P$ is assumed to be equal for all of the transmitters. By tuning the transmission power as well as the channel assignment, the SIR will be further improved. Replacing $P$ in Eqs. (23) and (24) by $P_m$, which indicate the transmission power level $m$, it becomes also tunable. The neurons, $x_{ijm}$, are defined as follows, when $x_{ijm} = 1$, the access point $i$ uses the channel $j$ with transmission power level $P_m$. By calculating in a similar way to the previous sections, the mutual interactions and the external magnetic field can be also calculated for solving the problem including transmission power control by the CIM [17].

### 4.3 Other Applications on Wireless Communication Systems

Optimization problems of the channel assignment and the transmission power allocation arise in various wireless communication systems. In a previous research, the CIM has been also applied to optimization of the Device-to-Device communications, with channel assignment and power allocation [17]. The CIM has also been applied to a scheduling problem in a distributed antenna system [18]. Those methods can be also realized by a similar scheme introduced in this paper. In Refs. [9], [10], a neural network based optimization scheme has been applied to a sensor network routing problem and a network load balancing problem, respectively. They can be also solved by the CIM in an ultra-high-speed by obtaining $J_{ij,kl}$ and $h_{ij}$ using Eqs. (21) and (22).

### 5. Conclusion

The CIM can search the solutions of combinatorial optimization problems in a quite high speed, hundreds microseconds to several milliseconds. The measurement-and-feedback type CIM [4], [5] can be applied to the larger problems even for dense or full mutual interactions in the corresponding Ising model. A previous research [8] have shown that the CIM has higher performance than the conventional quantum annealing machines [1] for such problems with dense mutual interactions.

In this paper, we have applied the CIM to the TSPs and optimization problems in wireless communication systems. Calculation to design the CIM for solving those target problems has been described in detail. The results of the real machine of the CIM have also been shown in the applications to the TSPs.

The real machine CIM used in the experiments can run 512 Ising spins with 8 bit precision of the mutual interactions. Development of larger scale CIM with more Ising spins is currently going on. By increasing the number of the Ising spins on the CIM, much larger scale combinatorial optimization problems will be also solved in very short time.

In communication systems, there are various resource assignment and scheduling problems. However, it is difficult to solve those problems and to tune the system in real time, especially in dynamically changing environments in communication traffics, wireless channels and so on. For such a real-time optimization, the CIM can be a candidate solution and to make it possi-
ble to keep the systems high performance.

The contents of this article is partially based on a previous publication Ref. [19].

References


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