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Electromagnetic Characteristics of Transverse Acousto-Optic Waveguide Device in Integrated Optics

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SUMMARY Among several optical devices in integrated optics, the fundamental characteristics of collinear optical switching devices have been studied about optical dielectric waveguides. Conventional waveguide-type acousto-optic (A-O) devices use collinear and longitudinal interactions with mode coupling based on the Bragg condition between optical waves and surface acoustic waves (SAW). Collinear A-O devices of the waveguide-type show sufficient performance for wavelength-selective switching with narrow bandwidths. However, in these collinear A-O devices, interaction time is several microseconds for 10 mm waveguide device length. In A-O devices of optical waveguides using transverse A-O interaction, where SAW propagates transversely to optical wave propagation direction, SAW propagation lengths needed for complete A-O interaction may become 10 μm and interaction time may be several nanoseconds. In this paper, fundamental characteristics of the transverse A-O interaction are studied as an electromagnetic boundary value problem. Refractive indices in optical waveguides induced by A-O effects with SAW are shown by sine functions. Wave field characteristics in periodic structures for transverse directions are analyzed by analytic method of Hill's equations for transverse spectral functions. Electromagnetic fields in regions with periodic structures are discussed by the Mathieu functions and the perturbation method. Dispersion characteristics of A-O eigen modes are studied for wavelengths of optical waves and SAW, with A-O coefficients.

key words: *A-O waveguide, SAW, mode conversion, Mathieu function, perturbation method*

1. Introduction

Integrated optics and optical devices consisting of active and passive waveguides have been rapidly developed recently. Among several optical devices, such as optical switches using A-O effects, the fundamental characteristics and application processing systems of collinear optical switching device using A-O effects, have been studied about optical dielectric waveguides on $LiNbO_3$ crystal substrates [1]–[3]. Conventional waveguide-type A-O devices use collinear interaction with mode coupling based on the Bragg condition between optical waves and SAW both propagating in the same direction. Collinear A-O devices of the waveguide-type show sufficient performance for wavelength selective switching with narrow bandwidths in case of low switching speed [4]–[7]. However, since the SAW propagation speed is very slow, comparing with the optical wave speed, and in these collinear A-O devices, the interaction time is several microseconds for 10 mm waveguide device length. In A-O waveguide devices using the transverse A-O interaction, where SAW propagates transversely and at the right angle to the optical wave propagation direction, SAW propagation lengths needed for complete A-O interaction may become 10 μm and interaction time may be several nanoseconds [8]–[10]. Ultrahigh-speed switching can be accomplished in A-O waveguide devices with transverse interaction of optical modes and SAW.

In this paper, the fundamental characteristics of transverse A-O interaction are studied as an electromagnetic boundary value problem. Refractive indices in optical waveguides induced by A-O effects of SAW are shown by sine functions in the transverse direction and yield wave equations with functional coefficients. Wave field characteristics in inhomogeneous media of periodic structures for transverse directions given by A-O effects due to SAW are analyzed by analytic method of Hill's equations for transverse spectral functions [11]. Electromagnetic fields in regions with periodic structures expressed by sine functions corresponding to SAW fields are shown by Mathieu functions with parameters and concerned with eigenvalues and a perturbation method.

By boundary conditions for electric and magnetic fields at boundaries between core and clad regions, eigen equations for eigen modes in the transverse-type A-O waveguides are derived. Dispersion characteristics of A-O eigen modes are shown by the perturbation method to Mathieu equation with A-O coefficients for wavelengths of optical waves and SAW. Switching and modulation properties due to propagation phase changes of the fundamental mode by A-O effects are shown. Based on these fundamental field characteristics of A-O waveguides, the mode coupling and switching in the transverse A-O waveguide devices consisting of coupled several optical waveguides controlled by SAW may be shown.

2. Optical Functional Waveguide with A-O Effects and Collinear Interaction

Interactions between SAW and optical waves are expressed by refractive index tensors. For A-O effects, physical relations among electric field E_k, electric displacement D_i, particle displacement u_j, stress tensor T_{ij} and strain tensor S_{kj} are expressed using physical coefficients of elastic coefficients C^E_{ijkl}, piezoelectric coefficients e_{kij}, dielectric constants ε_{ij}, A-O coefficients p_{ijkl}. Refractive index changes Δn_{il} due to strains of SAW are [7]

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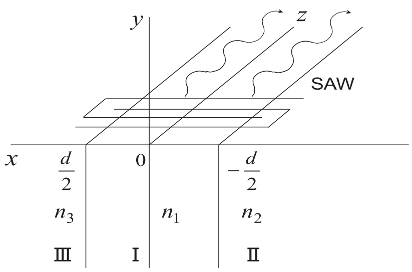

Fig. 1 Collinear A-O waveguide

$$\Delta n_{il} = -\frac{1}{2}\sum_{k,j}\frac{n_{ik}^2}{n_{il}}\left(\sum_{\alpha,\beta} p_{kj\alpha\beta}S_{\alpha\beta}\right)n_{jl}^2. \tag{1}$$

For the one-dimensional case,

$$\Delta n = -\frac{n^3}{2}pS. \tag{2}$$

Refractive index changes induced by the acoustic longitudinal wave with angular frequency Ω, wave number K and amplitude S_0, propagating in the z direction in homogeneous media are given, for the $x_1 = x$, $x_2 = y$ and $x_3 = z$ components, as

$$\Delta n_1 = -\frac{1}{2}n^3 p_{12}S_3, \quad \Delta n_3 = -\frac{1}{2}n^3 p_{11}S_3, \tag{3}$$

where $S_3 = S_0\sin(\Omega t - Kz)$ and $S_1 = S_2 = 0$.

For displacement vector $\boldsymbol{u}_a$ and electric frequency of SAW ω', using the double inner product of dyadics and tensors,

$$\Delta\varepsilon\,(\boldsymbol{r},\omega') = \boldsymbol{p} : \boldsymbol{s}(\boldsymbol{r},\omega'),$$
$$\boldsymbol{s} = b\boldsymbol{u}_a(\boldsymbol{r}_i, z, \omega')e^{-jK(\omega')z},$$

where b is the material constant of elastic properties.

Figure 1 shows the collinear and longitudinal optical waveguide of two-dimensional case with SAW propagating in the z direction. For A-O devices such as optical modulators, switches, couplers and separators using A-O effects, one or two A-O waveguides are combined.

General characteristics of active optical waveguide with A-O effects are shown by the following Fourier components of dielectric constants in the core and clad waveguide regions for several electric frequencies of SAW ω_i.

$$\Delta\bar{\varepsilon} = \sum_i \Delta\bar{\varepsilon}^{(i)}(\boldsymbol{r},t), \quad \Delta\varepsilon = \sum_i \varepsilon^{(i)}(\boldsymbol{r},\omega), \tag{4}$$

where

$$\Delta\varepsilon = \int_{-\infty}^{\infty}\Delta\bar{\varepsilon}(\boldsymbol{r},t)e^{-j\omega t}dt, \tag{5}$$
$$\Delta\bar{\varepsilon} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Delta\varepsilon e^{j\omega t}d\omega.$$

The optical electric fields in the waveguides are

$$E(\boldsymbol{r},\omega) = \int_{-\infty}^{\infty}\bar{E}(\boldsymbol{r},t)e^{-j\omega t}dt, \tag{6}$$
$$\bar{E}(\mathbf{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\boldsymbol{E}(\boldsymbol{r},\omega)e^{j\omega t}d\omega.$$

For several SAW multi-frequencies ω_i, the electric fields are described as

$$\boldsymbol{E}(\boldsymbol{r},\omega) = \sum\nolimits_i \boldsymbol{E}^{(i)}(\boldsymbol{r},\omega). \tag{7}$$

By linear A-O effects, the vector wave equations for electric fields in optical waveguides are

$$\nabla\times\nabla\times\boldsymbol{E}(\boldsymbol{r},\omega) - \mu\varepsilon\omega^2\boldsymbol{E}(\boldsymbol{r},\omega) =$$
$$\mu\Bigg[\int(-\omega'^2)\Delta\varepsilon(\omega')\boldsymbol{E}(\omega-\omega')d\omega' + \int(-2\omega')(\omega -$$
$$\omega')\Delta\varepsilon(\omega')\boldsymbol{E}(\omega-\omega')d\omega' +$$
$$\int(-(\omega-\omega')^2)\Delta\varepsilon(\omega')\boldsymbol{E}(\omega-\omega')d\omega'\Bigg]. \tag{8}$$

Optical frequencies are higher than SAW frequencies, $\omega' = \omega_i \ll \omega$ and then right hand term is

$$\mu\left[\int(-\omega^2)\Delta\varepsilon(\omega')\boldsymbol{E}(\omega-\omega')d\omega'\right]. \tag{9}$$

In active optical waveguide devices with A-O effects consisting of several optical waveguides (i), using eigen modes $\boldsymbol{\phi}_\alpha$ and eigenvalues β_α in a normal waveguide without A-O effects, we consider Green's dyadics

$$\boldsymbol{\Gamma}_{ij}(\boldsymbol{r},\boldsymbol{r}') = \sum_\alpha \frac{\boldsymbol{\phi}_\alpha(\boldsymbol{r},\omega)\boldsymbol{\phi}_\alpha(\boldsymbol{r}',\omega)}{M_\alpha(\boldsymbol{\omega})}\boldsymbol{e}^{-j\beta_\alpha(\omega)|z-z'|} \tag{10}$$

where $M_\alpha(\boldsymbol{\omega})$ is the normalization factor of eigen modes,

$$\int\boldsymbol{\phi}_\alpha(\boldsymbol{r}_t,\omega)\boldsymbol{\phi}_{\alpha'}(\boldsymbol{r}_t,\omega)d^2\boldsymbol{r}_t = \delta_{\alpha\alpha'}.$$

Electric fields $\boldsymbol{E}_i$ are generated by field source at the input. From the Green's formula, we have

$$\boldsymbol{E}_{totl} = \int\boldsymbol{\Gamma}_{ij}\int\mu\omega^2\Delta\varepsilon_j(\omega')\boldsymbol{E}_j(\omega-\omega')d\omega'dv' -$$
$$\int_{S_0}\int_{S_i}\boldsymbol{n}\cdot[(\nabla\times\boldsymbol{E}_{totl})\times\boldsymbol{\Gamma}_{ij} + \boldsymbol{E}_{totl}\times(\nabla\times$$
$$\boldsymbol{\Gamma}_{ij})]d\boldsymbol{S}'d\omega'. \tag{11}$$

The electric fields in active A-O waveguides can be expanded as the following field expansion series with amplitudes a_{α_i} by normal modes $\boldsymbol{\phi}_{\alpha_i}$

$$\boldsymbol{E}_i(\boldsymbol{r}_t,\omega) = \sum_\alpha a_{\alpha_i}(z,\omega)\boldsymbol{\phi}_{\alpha_i}(\boldsymbol{r}_t,\omega)e^{-j\beta_{\alpha_i}(\omega)z}. \tag{12}$$

Using integral operator $H = \int d\omega'$, from Eqs. (11) and (12), in A-O waveguides, mode coupling equations for mode amplitudes a_{α_i} by mode couplings and perturbations due to A-O effects are derived as

$$\frac{d\boldsymbol{a}}{dz} = HA\boldsymbol{a}, \quad \boldsymbol{a} = (a_1, a_2, \cdot\cdot\cdot\cdot a_n)^t, \tag{13}$$

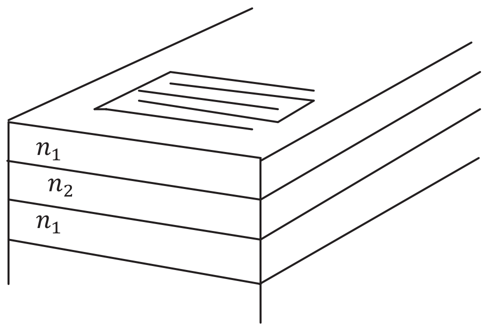

Fig. 2 Collinear A-O waveguide coupler

$$A_{ij} = (-j\zeta_{ij})e^{j\Delta k_{ij} z},$$

where the mode coupling coefficients A_{ij}, ζ_{ij} are

$$-j\zeta_{\alpha\alpha'}(z,\omega,\omega',\omega-\omega') = \frac{1}{M_\alpha}\int(\mu\omega^2)b\phi_{\alpha_i}(\boldsymbol{r}'_t,\omega)\boldsymbol{p}(\boldsymbol{r}'_t,z):\boldsymbol{u}_a(\boldsymbol{r}'_t,z,\omega')\times \phi_{\alpha'_i}(\boldsymbol{r}'_t,\omega-\omega')d^2\boldsymbol{r}'_t, \qquad (14)$$

and the phase differences concerned with the Bragg relations Δk_{ij} are

$$\Delta k_{ij}(\omega,\omega',z) = \beta_i(\omega) - \beta_j(\omega-\omega') - K(\omega',z). \qquad (15)$$

Using normalized mode amplitudes $b_s^{(i)}$ and normalized mode coupling coefficients $c_{ss'}$, the mode coupling equations are derived as,

$$\frac{db_0^{(i)}}{dz} = -j\beta_0^{(i)}b_0^{(i)} - jc_{01}^{(i)}b_1^{(i)} - j\sum_s c_{0s}^{(i,j)}b_s^{(i,j)},$$
$$\frac{db_1^{(i)}}{dz} = -j\beta_1^{(i)}b_1^{(i)} - jc_{10}^{(i)}b_0^{(i)} - j\sum_s c_{1s}^{(i,j)}b_s^{(i,j)}, \qquad (16)$$
$$\frac{db_s^{(i,j)}}{dz} = -j\beta_s^{(i,j)}b_s^{(i,j)} - jc_{0s}^{(i,j)}b_0^{(i)} - jc_{1s}^{(i,j)}b_1^{(i)}.$$

A collinear A-O waveguide coupler consisting of two waveguides with longitudinal SAW is shown in Fig. 2. Here, dominant mode amplitude $b_0^{(i)}$ is given by Eq. (17), when $b_s^{(i,j)} = 0$, and $c_{01}^{(i)} = c_{10}^{(i)} = c$ for the fundamental modes of $b_0^{(i)} =$ and $b_1^{(i)}$ in two waveguides.

$$\left|b_0^{(i)}\right|^2 = \frac{|c|^2}{|c|^2+|\Delta\beta|^2}\sin^2\!\left(\sqrt{|c|^2+|\Delta\beta|^2}\,z\right), \qquad (17)$$

where the phase relations of optical waves in two waveguides and SAW are

$$\Delta\beta = K - \Delta\beta_{opt}, \quad \Delta\beta_{opt} = \left|\beta_0^{(i)} - \beta_1^{(i)}\right|. \qquad (18)$$

The mode amplitudes $b_0^{(i)}$ show sharp wave filtering and separator characteristics controlled by SAW.

3. Structure of Transverse A-O Waveguide Device

In the transverse A-O waveguide shown in Fig. 3, optical

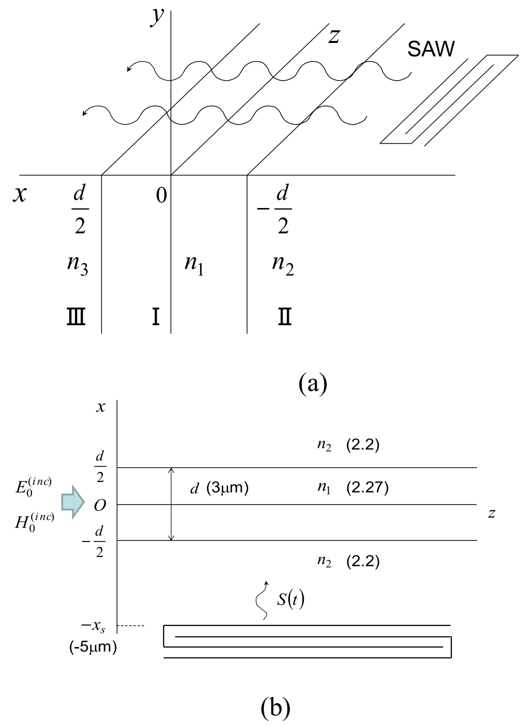

Fig. 3 Transverse A-O waveguide device

waves propagate in the longitudinal z direction and SAW propagates in the transverse x direction, where in the y direction the waveguide has uniform characteristics. Region I is the core part with refractive index n_1 and width d, and regions II and III are the clad parts with refractive indices n_2 and n_3. Transverse plane of the waveguide is the xy cross section. Strain $S(x,t)$ induced by SAW with velocity Ω/K and phase factor ϕ is given by

$$S(x,t) = S_0\sin(\Omega t - Kx - \phi). \qquad (19)$$

Refractive index changes in the homogeneous media of regions i with refractive indices n_i, using elasto-optic coefficients p_i, are

$$\Delta n_i = -\frac{1}{2}n_i^3 p_i S(x,t). \qquad (20)$$

When the dielectric constants of core and clad regions in the transverse A-O waveguide are $\tilde{\varepsilon}_1$, $\tilde{\varepsilon}_2$ and $\tilde{\varepsilon}_3$, wave numbers of optical waves in the transverse x direction change to $\beta_x \pm K$ by SAW with the wave numbers K and the propagation constants β_z in the longitudinal z direction are controlled to $\sqrt{\beta^2-(\beta_x \pm K)^2}$, where $\beta = \omega\sqrt{\mu\tilde{\varepsilon}}$ is the wave number of media without SAW. These characteristics of wave spectrum and dispersions are shown in Fig. 4.

Figure 5 shows the refractive index distributions due to A-O effects with SAW and dynamical characteristics of sinusoidal SAW responses in the core and clad regions of the transverse A-O waveguide.

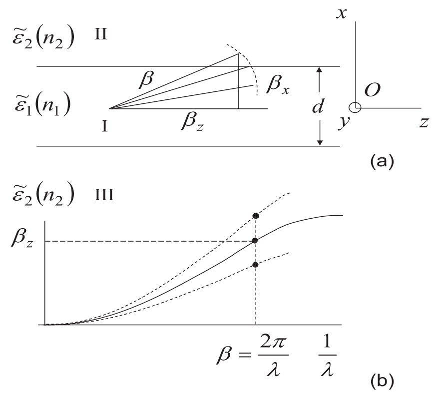

Fig. 4 Mode spectrum in A-O waveguide

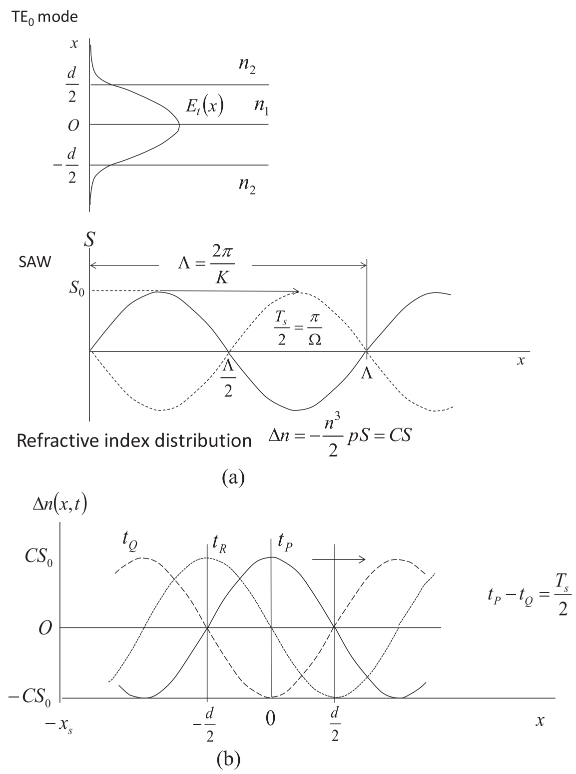

Fig. 5 Strain and refractive index distribution due to SAW
λ optical wavelength
Λ wavelength of SAW, T_s period of SAW

4. Electromagnetic Field in Transverse A-O Waveguide

Field and wave equations in the core (I) and clad (II), (III) regions of transverse A-O waveguides controlled by SAW are shown by changes of refractive indices and dielectric constants in each region as

$$\tilde{\varepsilon}_i = \varepsilon_i + \Delta\varepsilon_i, \ \ \tilde{\varepsilon}_i = \tilde{n}_i^2\varepsilon_0, \ \ \Delta\varepsilon_i = \Delta n_i^2\varepsilon_0,$$

$$\tilde{n}_1 = n_1 + \Delta n_1, \ \ \tilde{n}_2 \ (= \tilde{n}_3) = n_2 + \Delta n_2, \tag{21}$$

$$\Delta\varepsilon_i = -\Delta\varepsilon_{is}\sin(\Omega t - Kx - \phi), \tag{22}$$

where $\Delta\varepsilon_{is}$ is the magnitude of dielectric constant change, and the phase factor of SAW in initial state is ϕ. From the Maxwell equation, we have the vector wave equation as

$$\nabla\times\nabla\times\boldsymbol{E} - \mu\frac{\partial^2}{\partial t^2}\tilde{\varepsilon}\boldsymbol{E} = 0. \tag{23}$$

Hence,

$$\left(\nabla^2 - \mu\tilde{\varepsilon}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E} = \mu\Delta\varepsilon\frac{\partial^2\boldsymbol{E}}{\partial t^2} - \nabla(\nabla\ln\tilde{\varepsilon}\boldsymbol{E}). \tag{24}$$

From Eq. (21), in the region of A-O effects, the electric field is given as

$$\left(\nabla^2 - \mu\tilde{\varepsilon}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E} = -\mu\Delta\varepsilon_{is}\sin(\Omega t - Kx - \phi)\frac{\partial^2\boldsymbol{E}}{\partial t^2} - \nabla(\nabla\ln\tilde{\varepsilon}\boldsymbol{E}). \tag{25}$$

The magnetic field is given by, when the angular frequency of SAW Ω is smaller than the optical frequency ω,

$$\boldsymbol{H} = \frac{j}{\mu\omega}\nabla\times\boldsymbol{E}, \quad \omega \gg \Omega. \tag{26}$$

Here, the electromagnetic fields $\boldsymbol{E}^{(\mathrm{i})}(x,z,t)$ in the core and clad regions, (I) and (II), for the uniform field in the y-direction are shown as, if the propagation factor of the z direction is $e^{-j\beta_z z}$ and the time factor is $e^{j\omega t}$,

$$\boldsymbol{E}^{(i)}(x,z,t) = \boldsymbol{E}_t^{(i)}(x)e^{-j\beta_z z + j\omega t}, \tag{27}$$

where

$$E_y(x,z,t) = E_t(x)e^{j\omega t - j\beta_z z},$$
$$H_x(x,z,t) = H_t(x)e^{j\omega t - j\beta_z z}.$$

The transverse electric fields satisfy, using the frequency relation of $\omega \gg \Omega$

$$\left[\frac{\partial^2}{\partial x^2} + \mu\omega^2(\varepsilon_i + \Delta\varepsilon_{is}\sin(Kx+\phi-\Omega t)) - \beta_z^2\right]\boldsymbol{E}_t^{(i)}(x) = 0, \tag{28}$$

when the phase relation is $\phi - \Omega t = \varphi(t)$,

$$\left[\frac{\partial^2}{\partial x^2} + \mu\omega^2(\varepsilon_i + \Delta\varepsilon_{is}\sin(Kx+\varphi)) - \beta_z^2\right]\mathbf{E}_t^{(i)}(x) = 0. \tag{29}$$

In Eq. (29), we define the parameters of SAW and waveguides W, η, a_i, q_i, following as, for y polarization and $\mathbf{E}_t^{(i)} = E_y^{(i)}\mathbf{i}_y$, defining new coordinate x',

$$Kx + \varphi - \frac{\pi}{2} = 2\eta = Kx',$$

$$a_i = W^2\left(\mu\omega^2\varepsilon_i - \beta_z^2\right) = \frac{4\left(\mu\omega^2\varepsilon_i - \beta_z^2\right)}{K^2},$$

$$-2q_i = W^2\mu\omega^2\Delta\varepsilon_{is} = \frac{4\mu\omega^2\Delta\varepsilon_{is}}{K^2}, \tag{30}$$

$$\beta_i^2 = \mu\omega^2\tilde{\varepsilon}_i = \tilde{n}_i^2\varepsilon_0.$$

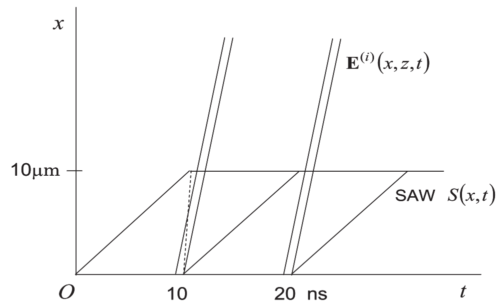

Fig. 6 Space-time interaction of optical modes and SAW

Here, for wavelength Λ of SAW, we define K and W, as

$$W = \frac{2}{K} = \frac{\Lambda}{\pi}, \quad K = \frac{2}{W} = \frac{2\pi}{\Lambda} \quad \text{and} \quad \frac{d}{dx} = \frac{K}{2}\frac{d}{d\eta} = \frac{1}{W}\frac{d}{d\eta}$$

Equation (29) of transverse electric fields $\mathbf{E}_t^{(i)}(x)$ is reduced to the following Mathieu equation with parameter q and eigenvalues a for η coordinate variable

$$\left[\frac{d^2}{d\eta^2} + (a_i - 2q_i\cos 2\eta)\right]E_t^{(i)}(\eta) = 0. \tag{31}$$

In the transverse A-O waveguide, the propagation characteristics of optical waves and SAW are shown, for distance x, and time t, in Fig. 6.

The propagation velocity of SAW $V = \dfrac{\Omega}{K}$ is smaller than those of optical modes $v = 1/\sqrt{\varepsilon_{eq}\mu}$, and optical waves pass through the transverse cross section of SAW xy plane with short-time interactions and high-speed responses. Where ε_{eq} is equivalent dielectric constant of optical modes, $\dfrac{\beta_z}{\beta} = \sqrt{\varepsilon_{eq}}$.

The electric fields in the core (I) and clad (II), (III) regions, for y polarization, $\mathbf{E}_t^{(i)}(x) = \mathbf{i}_y E_y^{(i)}(x)$, and the z component of magnetic fields $H_z^{(i)}$ are, using Mathieu functions and coefficients $C_\nu^{(i)}$, $S_\nu^{(i)}$

$$E_y^{(i)}(\eta) = \sum_{\nu=0}\left(C_\nu^{(i)} ce_\nu(\eta, q_i) + S_\nu^{(i)} se_\nu(\eta, q_i)\right),$$
$$H_z^{(i)}(\eta) = \frac{1}{j\omega\mu}\frac{1}{W}\sum_{\nu=0}\left[C_\nu^{(i)} ce'_\nu(\eta, q_i) + S_\nu^{(i)} se'_\nu(\eta, q_i)\right]. \tag{32}$$

Here, for small q parameter cases, we have Mathieu functions based on sine and cosine functions as

$$a_i = \nu^2 + \sum_{r=0}^{\infty}\alpha_r q_i^r, \quad \alpha_2 = \frac{1}{2(\nu^2 - 1)}, \tag{33}$$
$$ce_\nu(\eta, q) = \cos\nu z - \frac{1}{4}q\left[\frac{\cos(\nu+2)\eta}{(\nu+1)} - \frac{\cos(\nu-2)\eta}{(\nu-1)}\right] + O(q^2),$$
$$se_\nu(\eta, q) = \sin\nu z - \frac{1}{4}q\left[\frac{\sin(\nu+2)\eta}{(\nu+1)} - \frac{\sin(\nu-2)\eta}{(\nu-1)}\right] + O(q^2).$$

From the continuity conditions of electric and magnetic fields at boundaries between core and clad, $x = \pm(d/2)$, when refractive indices of clad regions II and III, $\tilde{n}_2 = \tilde{n}_3$, using coefficients $C_\nu^{(1)}$, $S_\nu^{(1)}$, $C_\nu^{(2)}$, $S_\nu^{(2)}$ and defining, for boundaries of η coordinates,

$$\eta_\pm = \frac{K}{2}\left(\pm\frac{d}{2}\right) + \frac{1}{2}\left(\varphi - \frac{\pi}{2}\right),$$

we have the eigen equation and derive the dispersion characteristics of eigen modes giving propagation constants.

As a simple case, we next consider the transverse A-O effects in the core region, without A-O effects in the clad regions. In case of $\Delta\varepsilon_2 = \Delta\varepsilon_3 = 0$, and $\tilde{\varepsilon}_1 = \varepsilon_1 + \Delta\varepsilon_1$, $\varepsilon_1 > \varepsilon_2 = \varepsilon_3$, $\Delta\varepsilon_1 = -\Delta\varepsilon_{1s}\sin(\Omega t - Kx - \phi)$, where only the core region has A-O effects, we discuss simple case of mode conversions due to A-O effects in the core region and no A-O effects in the clad region. For $\varphi = \pi/2$, the TE symmetric (even) mode for y polarization, the electric and magnetic fields $E_y^{(1)}$, $E_y^{(2)}$, $H_z^{(1)}$, $H_z^{(2)}$, are, using Mathieu functions and $C_\nu^{(1)}$, $C_\nu^{(2)}$ in the core and the clad regions,

$$E_y^{(1)} = C_\nu^{(1)} ce_\nu(\eta, q_1)e^{-j\beta_z z},$$
$$H_z^{(1)} = -\frac{j}{\omega\mu}\frac{1}{W}C_\nu^{(1)} ce'_\nu(\eta, q_1)e^{-j\beta_z z}, \quad |x| \le \frac{d}{2}, \tag{34}$$
$$E_y^{(2)} = C_\nu^{(2)} e^{-j\beta_z z}e^{\pm\alpha_x^{(2)}x}, \quad H_z^{(2)} = \frac{\pm\alpha_x^{(2)}}{\omega\mu}jC_\nu^{(2)}e^{-j\beta_z z}e^{\pm\alpha_x^{(2)}x},$$
$$|x| \ge \frac{d}{2}$$

Here, in Eq. (33), we define $a_{\mathrm{i}} = a_1$, $\alpha_{\mathrm{r}} = \alpha_{r1}$, $q_{\mathrm{i}} = q_1$, and for spectrum parameters

$$-\alpha_x^{(i)2} = \beta_i^2 - \beta_z^2 = \beta_x^{(i)2}, \quad \alpha_x^{(2)} = \alpha_x^{(3)},$$

and coefficients $C^{(2)} = C^{(3)}$.

The boundary condition of the electric and magnetic fields E_y and H_z at the boundary interface at, $x = \pm\dfrac{d}{2}$ are

$$C_\nu^{(1)} ce_\nu\left(\eta^{(d)}, q_1\right) = C_\nu^{(2)} e^{-\alpha_x\frac{d}{2}},$$
$$\frac{1}{W}C_\nu^{(1)} ce'_\nu\left(\eta^{(d)}, q_1\right) = \alpha_x C_\nu^{(2)} e^{-\alpha_x\frac{d}{2}},$$
$$-c\,e'_\nu(\eta, q_1)\Big|_{\eta=+\frac{K}{4}d} = +c\,e'_\nu(\eta, q_1)\Big|_{\eta=-\frac{K}{4}d}, \tag{35}$$

where

$$\eta^{(d)} = \frac{K}{4}d \quad \text{and} \quad \alpha_x = \alpha_x^{(2)}.$$

From the field continuity conditions of Eq. (35), we have mode dispersion relations of TE symmetric modes as

$$\frac{C_\nu^{(1)}}{C_\nu^{(2)}} = \frac{e^{-\alpha_x\frac{d}{2}}}{ce_\nu\left(\eta^{(d)}, q_1\right)} = \frac{\alpha_x e^{-\alpha_x\frac{d}{2}}}{\frac{1}{W}ce'_\nu\left(\eta^{(d)}, q_1\right)}, \tag{36}$$
$$\frac{c\,e'_\nu\left(\eta^{(d)}, q_1\right)}{c\,e_\nu\left(\eta^{(d)}, q_1\right)} = -\alpha_x W,$$

where

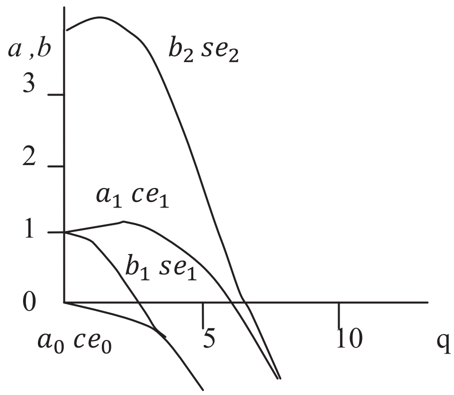

Fig. 7 Eigen characteristics a and b of TE modes for Mathieu functions ce_n and se_n

$$\beta_z^2 - \alpha_x^2 = \beta_2^2, \quad \alpha_x W = \sqrt{\left(\beta_1^2 - \beta_2^2\right)W^2 - 1},$$
$$\beta_z^2 + \left(\frac{1}{W}\right)^2 = \beta_1^2.$$

The propagation constants $\beta_z^{(\nu)}$ of the optical guided ν modes and dispersion characteristics derived by solutions of eigen equation of Eq. (36) for Mathieu functions will be concretely discussed in a subsequent paper to be published in the future. The eigen characteristics of Mathieu functions yield parameter relations of q and a, b concerned with the TE symmetric modes as shown in Fig. 7.

The parameters a and b are represented, for the Mathieu functions ce_n and se_n, as eigen values a in Eq. (33). Since $\Delta\varepsilon$ and q are very small, the perturbation method to the Mathieu equations is useful in A-O waveguides.

5. Eigen Mode Characteristics by Perturbation Method to Mathieu Equation

The propagation constants and the eigen mode fields of A-O waveguides are derived by the perturbation method to solutions of the Mathieu equations, Eqs. (29) and (31), with A-O effects based on field expansions by the normal eigen modes in slab waveguides satisfying the boundary condition without A-O effects.

The wave fields controlled by A-O effects, $\boldsymbol{E}_y^{(i)}(x)$ in Eq. (29) are expanded by the eigen mode functions $\Phi_m = \Psi_m(x)e^{-j\beta_\Phi(m)z}$ in the normal waveguides without A-O effects of, $\tilde{\varepsilon}_i = \varepsilon_i$, $\Delta\varepsilon_i = 0$ as follows.

$$E_y^{(i)}(x) = \sum_m a_m(\beta_z)\Psi_m(x). \tag{37}$$

From Eq. (29), using integration in the x cross section, we have the following linear equation for amplitudes a_m

$$\int \sum_m a_m(\beta)\Psi_m\Psi_{m'}\left[\left(\beta_z^2 - \beta_\Phi^2\right) - \omega^2\mu(\tilde{\varepsilon}_i - \varepsilon_i)\right]dx = 0. \tag{38}$$

Defining the mode perturbation and mode conversion factors with the mode normalizations as,

$$K_{mm'} = -\omega^2\mu \int (\tilde{\varepsilon}_i - \varepsilon_i)\Psi_m\Psi_{m'}dx\frac{1}{N_{mm'}}, \tag{39}$$
$$N_{mm'} = \int \Psi_m\Psi_{m'}dx$$

and constituting matrix and vector $\boldsymbol{D}$, $\boldsymbol{\zeta}$, $\boldsymbol{a}$, of matrix and vector components, as follows,

$$\zeta_{2s-1,2t-1} = K_{2s-1,2t-1},$$
$$\zeta_{2s,2t} = K_{2s,2t},$$
$$D_{2s-1,2t-1} = D_{2s,2t} = \beta_\Phi^2 \quad (s = t)$$
$$= 0 \quad (s \neq t).$$

we have the following linear equation

$$\left[\left(\beta^2\boldsymbol{I} - \boldsymbol{D}\right) + \zeta\right]\boldsymbol{a} = 0. \tag{40}$$

From Eq. (40), the eigen characteristic equation of the eigen modes in the A-O waveguide, using components $H_{p\gamma}$ of matrix $\boldsymbol{D} - \boldsymbol{\zeta}$, by the perturbation method for small $\Delta\varepsilon$, we have eigen vectors $\boldsymbol{\alpha}_\gamma$ and eigen vectors β_γ^2

$$\boldsymbol{\alpha}_\gamma = \boldsymbol{a}_\gamma - \sum_{p\neq\gamma}\frac{H_{p\gamma}}{\beta^2 - H_{pp}}\boldsymbol{a}_p + \cdots,$$
$$\beta_\gamma^2 = H_{\gamma\gamma} + \sum_p \frac{H_{\gamma p}H_{p\gamma}}{\beta^2 - H_{pp}} + \cdots. \tag{41}$$

The propagation constants of the γth mode in the A-O waveguide are

$$\beta_\gamma = \beta_\Phi^2 - K_{\gamma\gamma'}. \tag{42}$$

Here, the media perturbation term due to A-O effects is

$$\tilde{\varepsilon}_1 - \varepsilon_1 = \Delta\varepsilon = \Delta\varepsilon_s \sin(Kx + \varphi) \tag{43}$$

For the fundamental even TE_0 mode of amplitude C_0 as

$$\Psi_{(0)} = C_0 \cos\beta_{1x}x,$$

the perturbation factors concerned with A-O effects are given by

$$K_{00'} = -\omega^2\mu\Delta\varepsilon_s\frac{1}{d}\int_{-d/2}^{d/2}\sin(Kx+\varphi)\cos(\beta_{1x}x)\cos(\beta'_{1x}x)dx$$
$$= -\omega^2\mu\Delta\varepsilon_s C_{00}, \tag{44}$$

where the mode coupling and perturbation factors C_{00} are

$$C_{00} = \frac{1}{d}\int_{-d/2}^{d/2}\sin(Kx + \varphi)\cos(\beta_{1x}x)\cos(\beta'_{1x}x)dx. \tag{45}$$

When the x component of wave spectrum is $\beta_{1x} = \beta'_{1x}$, we have

$$C_{00} = \frac{1}{2}\sin\varphi\left[\frac{\sin\left(\frac{dK}{2}\right)}{\left(\frac{dK}{2}\right)} + \frac{1}{2}\sum_{w=\pm1}\frac{\sin\left(\frac{(K+2w\beta_{1x})d}{2}\right)}{\left(\frac{(K+2w\beta_{1x})d}{2}\right)} + \frac{\delta}{2}\right] \tag{46}$$

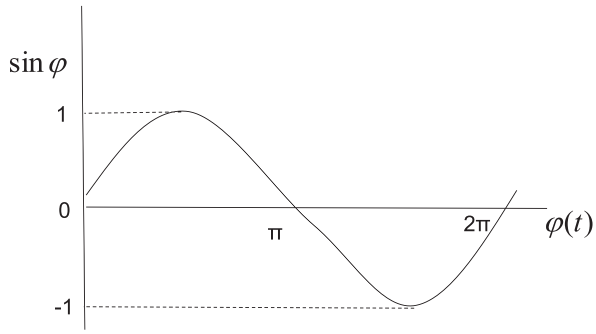

Fig. 8 Dynamic phase properties of controlling SAW
$\varphi(t) = \phi - \Omega t \quad \Lambda = \frac{2\pi}{K} \quad T_s = \frac{2\pi}{\Omega}$

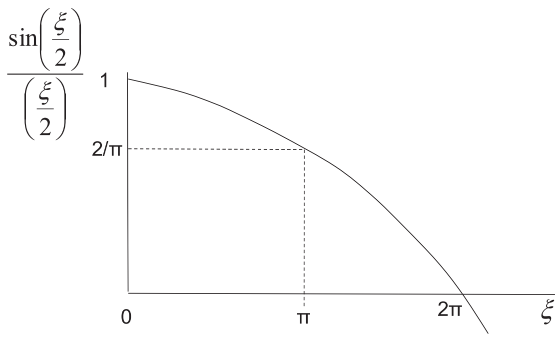

Fig. 9 Mode conversion factors due to A-O effects of SAW
$\xi = aK, a(K \pm 2\beta_{1x})$

where, $K = 2\beta_{1x}$ for $w = 1$, $\delta = 1$ and $K \neq 2\beta_{1x}$ for $w = \pm 1$, $\delta = 0$.

From Eqs. (42) and (44), the propagation constants of perturbed eigen modes due to A-O effects in the A-O waveguides are given by, for the fundamental dominant mode,

$$\beta'^2_{(0)} = \beta^2_{(0)} - K_{00} = \beta^2_{(0)} + \omega^2 \mu \, \Delta\varepsilon_s C_{00}. \tag{47}$$

These evaluation yield phase characteristics of the propagation modes. The transverse field distributions of perturbed eigen modes in A-O waveguides are shown as,

$$\alpha_{(0)} \cong a_{(0)}, \quad \Psi'_{(0)} \cong \Psi_{(0)} \tag{48}$$

for the fundamental dominant mode, in the first approximation, when $\Delta\varepsilon$ due to A-O effects is small.

Figures 8 and 9 show the dynamic phase properties of controlling SAW and the mode conversion factors due to the A-O effects of SAW, discussed in Eq. (47).

The transverse spectrum factors β_{1x} in the normal waveguides are analytically given by V values, from the eigen equation of the fundamental even TE mode,

$$\tan \beta_{1x} \frac{d}{2} = \frac{\alpha_{2x}}{\beta_{1x}}.$$

Near cutoff, defining ζ as

$$V = \sqrt{n_1^2 - n_2^2}\, \beta_0 \frac{d}{2}, \quad \zeta = V - m\pi,$$

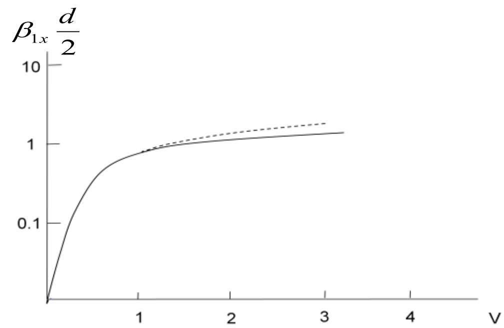

Fig. 10 Transverse spectrum β_{1x} of dominant TE mode in normal slab waveguide
dot line: approximation, solid line: exact

we have approximately the spectrum factors as follows,

$$\alpha_{2x} = \frac{1}{d}\left(\sqrt{4V^2+1} - 1\right),$$
$$\beta_{1x} = \frac{\sqrt{2}}{d}\left(\sqrt{4V^2+1} - 1\right)^{1/2},$$
$$\beta_z = \sqrt{(n_2\beta_0)^2 - \frac{2}{d^2}\left(1 + 2V^2 - \sqrt{4V^2+1}\right)}.$$

and the transverse spectrum factors β_{1x} are shown in Fig. 10.

The phase characteristics Φ of the fundamental guided even TE_0 mode perturbed by the A-O effects of SAW are given by, for propagation length z,

$$\Phi = (\beta_z + \Delta\beta_z)z, \quad \Delta\Phi = \Delta\beta_z z, \tag{49}$$

where $\Delta\Phi$ is the phase modulation due to the A-O effects of SAW in the A-O waveguide.

From Eq. (47), $\Delta\Phi$ is evaluated, using refractive index perturbation Δn_s and refractive index of the core in waveguide n_1, by perturbation factor C_{00} of Eq. (46), when the effective wavelength of the fundamental TE mode in the normal waveguide without the A-O effects is $\lambda_{\text{eff}} = 2\pi/\beta_z$, as follows

$$\Delta\Phi = \beta_z \frac{\Delta n_s}{2n_1} C_{00} z = \frac{\pi}{\lambda_{eff}} \frac{\Delta n_s}{n_1} C_{00} z \tag{50}$$

For example, when the waveguide parameters are $n_1 = 2.27$, $n_2 = 2.20$, $d = 3$ μm, $\lambda = 1$ μm, and V = 3.4, for $\Delta n_s = 10^{-3}$, $C_{00} = 3/4$, and $z = 1$ mm, we have $\Delta\Phi = 2.35$ (radian). This typical example provides excellent technical data for accomplishment of high-speed signal switching and processing.

6. Transverse A-O Wave Separator

Optical transverse A-O couplers and wave separators consisting of parallel waveguides with transversely propagating SAW shown in Fig. 11, can be discussed, based on the eigen mode characteristics of the transverse A-O waveguide. When one waveguide has the A-O effects, the propagation

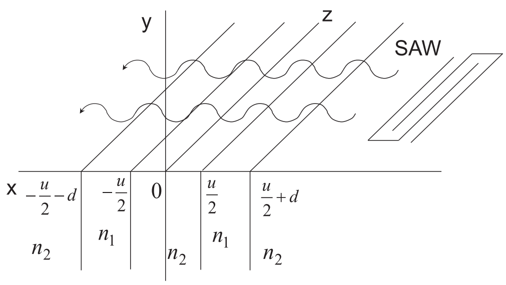

Fig. 11 Transverse A-O wave separator

constants are controlled by SAW for wave separation and wave filtering given in Eq. (17).

7. Conclusions

Transverse A-O waveguides with SAW have high efficient and high-speed switching characteristics. Electromagnetic field characteristics in the transverse A-O waveguides with refractive indices transversely controlled by SAW are studied by eigen function expansion methods for Hill's equations. Field characteristics are discussed by the Mathieu function expansion method and the perturbation method for the core and clad regions with the A-O effects due to SAW. Eigen equations and eigen modes are shown, using field expressions of Mathieu functions, by the field boundary condition and the perturbation method based on the eigen modes in the normal waveguides without the A-O effects. The dispersion characteristics for wave numbers and wavelengths of optical waves and SAW are shown. Transverse A-O wave couplers and wave separators are discussed.

References

[1] T. Kondo, Y. Miyazaki, and Y. Akao, "Optical tunable switched directional couplers consisting of two thin-film waveguides using surface acoustic waves," Jpn. J. Appl. Phys., vol.17, no.7, pp.1231–1243, 1978.
[2] N. Goto and Y. Miyazaki, "Integrated optical multi-/demultiplexer using acoustooptic effect for multiwavelength optical communications," IEEE J. Sel. Areas Commun., vol.8, no.6, pp.1160–1168, 1990.
[3] N. Goto and Y. Miyazaki, "Multi-wavelength separation characteristics in integrated acoustooptic devices for photonic switching system," Jpn. J. Appl. Phys., vol.40, no.5B, pp.3762–3767, 2001.
[4] N. Goto and Y. Miyazaki, "FDTD analysis of wavelength-selective characteristics in weighted acoustooptic switches for wavelength-division-multiplexed photonic routing," Jpn. J. Appl. Phys., vol.43, no.5B, pp.2958–2964, 2004.
[5] C.H. Von Helmolt, "Integrated optic strip waveguides driven by surface acoustic waves," J. Opt. Commun., vol.2, no.4, pp.142–154, 1981.
[6] K. Tada and H.S. Hinton, Photonic Switching II, Springer-Verlag, Heidelberg, 1990.
[7] Y. Miyazaki, "Non-linear interaction of optical modes with multi-electromagnetic waves in optical functional devices," IEEJ Trans. EIS, vol.118, no.1, pp.99–104, 1998.
[8] Y. Miyazaki, "Electromagnetic eigen field characteristics of acousto-optic waveguides with SAW," Proc. PIERS Taipei, pp.129–134, 2013.
[9] Y. Miyazaki, "Electromagnetic field characteristics of transverse mode couplings in acousto-optic waveguide devices," Proc. Int. Symp. EMT, URSI Comm.B, 23PM3E-03, pp.755–758, Hiroshima, 2013.
[10] Y. Miyazaki, "Optical modes in dielectric thin film fiber with convex surface by conformal mapping technique," Trans. IECE Jpn., vol.E59, no.10, pp.1–6, 1976.
[11] N.W. McLachlan, Theory and Application of Mathieu Functions, Dover Pub., New York, 1964.

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