

Mode-Matching Analysis for Circular and Annular Aperture Scattering

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SUMMARY The mode-matching applications to scattering from circular and annular apertures in a thick perfectly conducting plane are reviewed. The Hankel and Weber transforms are utilized to solve the boundary-value problems of circular and annular apertures. Simple electrostatic problems are presented to illustrate the mode-matching method in terms of the Hankel and Weber transforms. Various types of Weber transform are discussed with boundary-value problems. Electromagnetic radiation and scattering from circular and annular aperture geometries are summarized. The utility of the mode-matching method in circular and annular aperture scattering is emphasized.

key words: mode-matching method, circular and annular apertures, electromagnetic scattering, eigenfunction expansions

1. Introduction

Circular and annular apertures in a conducting plane are important geometries in electromagnetic wave scattering, antennas, microwave guiding structures, and electromagnetic interference. Due to their important applications, many numerical and analytical approaches have been utilized to understand electromagnetic wave interactions with circular and annular aperture structures. For instance, the method of moments was used in [1] to study electromagnetic scattering by a conducting screen perforated with circular holes. The method of moments was also used in [2] to investigate the current behavior along a wire penetrating a circular aperture. Since the geometry of circular and annular apertures in a conducting plane is a canonical structure in the cylindrical coordinates (ρ, ϕ, z) , it is possible to solve the problem with the technique of separation of variables and the mode-matching method in cylindrical coordinates. The mode-matching method utilizes eigenfunction expansions in cylindrical coordinates. The eigenfunction expansions such as the Hankel and Weber transforms are needed in the open domain, whereas Bessel function series are used to represent the field in the closed domain. Since the eigenfunction expansions are used in the mode-matching method to express the fields, the orthogonality of eigenfunctions is utilized to further simplify expressions when the boundary conditions are enforced. Because of the eigenfunction expansions, it is possible to obtain robust and numerically efficient solu-

tions. The mode-matching method yields rigorous, analytic solutions as compared with other numerical approaches.

The purpose of the present paper is to review how the mode-matching method can be utilized to solve the boundary-value problems of circular and annular apertures in a conducting plane. Using the mode-matching method, it is possible to analytically solve some very important problems of circular and annular geometries in microwave regime. The mode-matching method utilizes many techniques including the Fourier series, Hankel transform, Weber transform, residue calculus, and superposition. In particular, the Hankel transform and Weber transform play an important role in the mode-matching method to solve the boundary-value problems of scattering from circular and annular apertures in a thick conducting plane of infinite extent. For the sake of discussion, we will first summarize Hankel transform applications, and then will discuss Weber transform applications.

2. Hankel Transform Applications

The Hankel transform is an effective tool for describing the scattered field in the half-space above circular and annular apertures in a conducting plane of infinite extent. The Hankel transform has been widely used for the analysis of various circular and annular scattering. We begin with the case of circular apertures.

2.1 Circular Apertures

The basic idea of solving circular aperture scattering problems is to represent the fields in eigenfunction expansions. For the sake of discussion, we consider a boundary-value problem of electrostatic potential penetration into a circular aperture in a thick conducting plane of infinite extent, as shown in Fig. 1(a). The cross-sectional view of Fig. 1(a) is illustrated in Fig. 1(b). We note that a complete analysis of this problem is available in [3]. In region (I) ($z \geq 0$) the primary potential $\Phi^p = z$ impinges on a circular aperture with radius a and depth d in a thick conducting plane at zero potential. The total potential in region (I) consists of the primary potential Φ^p and the scattered potential Φ^s . The electrostatic scattered potential Φ^s is governed by Laplace's equation

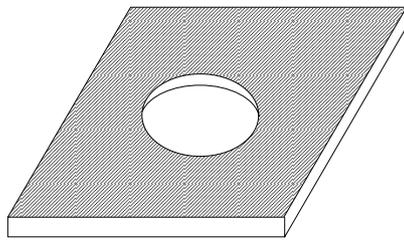
$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial \Phi^s}{\partial \rho} \right) + \frac{\partial^2 \Phi^s}{\partial z^2} = 0 \quad (1)$$

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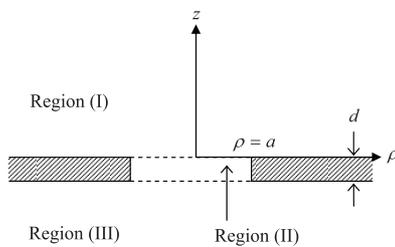
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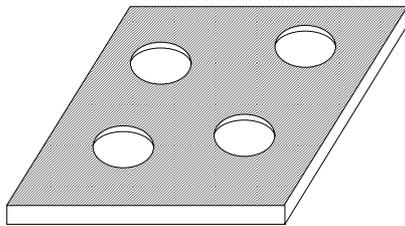
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(a)



(b)



(c)

Fig. 1 (a) Circular aperture in a thick conducting plane of infinite extent. (b) Cross-sectional view of circular aperture in a thick conducting plane of infinite extent. Regions (I) ($z \geq 0$), (II) ($-d \leq z \leq 0$ and $\rho \leq a$), and (III) ($z \leq -d$) represent the upper half-space, circular aperture, and lower half-space, respectively. (c) Multiple circular apertures in a thick conducting plane.

In view of the Hankel transform

$$\tilde{f}(\zeta) = \int_0^{\infty} f(\rho) J_0(\zeta \rho) \rho d\rho \quad (2)$$

$$f(\rho) = \int_0^{\infty} \tilde{f}(\zeta) J_0(\zeta \rho) \zeta d\zeta \quad (3)$$

it is possible to assume

$$\Phi^s = \int_0^{\infty} \tilde{\Phi}^s(\zeta) J_0(\zeta \rho) e^{-\zeta z} \zeta d\zeta \quad (4)$$

where $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind. The electrostatic potential within the circular aperture in region (II) is

$$\Phi^d = \sum_{n=1}^{\infty} [b_n \sinh k_n(z+d) + c_n \cosh k_n(z+d)] \times J_0(k_n \rho) \quad (5)$$

where $J_0(k_n a) = 0$ determines the parameters k_n . The transmitted potential in region (III) is

$$\Phi^t = \int_0^{\infty} \tilde{\Phi}^t(\zeta) J_0(\zeta \rho) e^{\zeta(z+d)} \zeta d\zeta \quad (6)$$

We apply the Hankel transform to the continuity of the potential at $z = 0$

$$\Phi^p|_{z=0} + \Phi^s|_{z=0} = \begin{cases} \Phi^d|_{z=0} & \text{for } \rho < a \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Then, we get a $\tilde{\Phi}^s(\zeta)$ representation in series of the discrete modal coefficients b_n and c_n . Applying the Bessel function orthogonality to the boundary condition at $z = 0$ for $\rho < a$

$$\frac{\partial}{\partial z} [\Phi^p + \Phi^s]_{z=0} = \frac{\partial}{\partial z} [\Phi^d]_{z=0} \quad (8)$$

yields a set of simultaneous infinite series equations for the discrete modal coefficients b_n and c_n . It is possible to obtain another equation for the discrete modal coefficients b_n and c_n from the boundary conditions at $z = -d$. After truncating infinite series, computations are performed to solve a set of simultaneous equations for the discrete modal coefficients. The main gist of mode-matching method is to obtain a series solution in terms of the discrete modal coefficients. Thanks to the eigenfunction expansions, the series solution converges fast.

In what follows, we will present various circular aperture scattering problems using the mode-matching method. The problems of electrostatic, magnetostatic, acoustic, and electromagnetic wave scattering from a circular aperture were solved in [3]–[6]. Note that the electrostatic, magnetostatic, acoustic, and electromagnetic problems are all governed by Helmholtz's equation, (Laplace's equation is considered as a static limit form of Helmholtz's equation). In particular, electromagnetic scattering from a circular aperture in a conducting plane finds practical applications in scattering theory and in electromagnetic compatibility-related problems [6].

The extension of a single aperture to multiple apertures is straightforward within the framework of mode-matching method. Electromagnetic scattering from multiple circular apertures finds important applications in frequency-selective surfaces, antennas, and electromagnetic interferences. Figure 1(c) illustrates multiple circular apertures in a thick conducting plane. The problems of electrostatic, magnetostatic, acoustic, and electromagnetic wave scattering from double/multiple circular apertures were solved in [7]–[12]. In particular, the electromagnetic wave scattering from multiple circular apertures in a thick perfectly conducting plane was considered in [12] using the power orthogonality. Computations indicate that the mode-matching solutions are indeed robust and accurate for practical applications. Hence

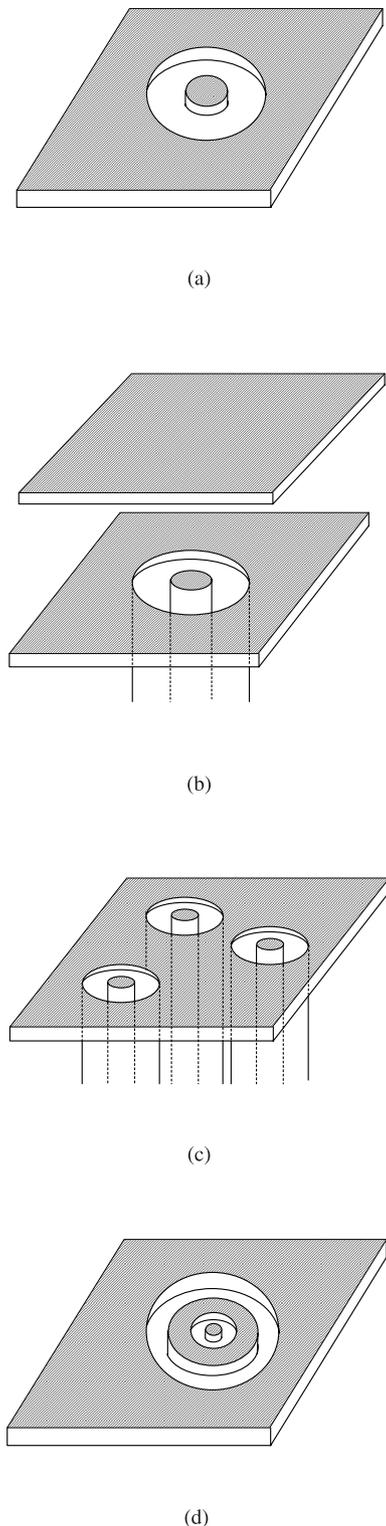


Fig. 2 (a) Annular aperture in a thick conducting plane of infinite extent. (b) Open-ended coaxial cable radiating into half-space. (c) Open-ended coaxial cable array. (d) Concentric annular apertures in a thick conducting plane.

the mode-matching solutions provide an efficient means to estimate electrostatic, magnetostatic, acoustic, and electromagnetic wave scattering from circular apertures in a thick

conducting plane.

2.2 Annular Apertures

The problem of scattering from annular apertures is important in microwave and millimeter wave device modeling. Figure 2(a) illustrates a single annular aperture in a thick conducting plane. When dimensions of annular apertures are small compared to the wavelength, a static assumption is applicable in microwave regime. Polarizabilities of small apertures are often used to characterize wave transmission and scattering in low-frequency regimes. Polarizabilities of annular apertures were extensively considered in [13]–[15] using the mode-matching method. When an open-ended coaxial cable excites waveguide structures, radiation and scattering from an annular aperture become an important problem. Figure 2(b) shows an open-ended coaxial cable radiating into a layered half-space. This problem was rigorously solved in [16], [17] using the Hankel transform and residue calculus. Radiation from an open-ended coaxial cable of Fig. 2(b) can be extended to the case of multiple, open-ended coaxial cables of Fig. 2(c). The problem of coupling between open-ended coaxial cable array of Fig. 2(c) was considered in [18], [19].

The electromagnetic scattering problems of concentric annular apertures are important in microwave device modeling and microwave antenna design. Figure 2(d) illustrates multiple annular apertures in a thick conducting plane. Radiation and scattering from various multiple annular slot structures were investigated in [20]–[22].

3. Weber Transform Applications

Although the Hankel transform is valuable to handle boundary-value problems in many areas of electromagnetic theory, its uses are restricted to the case of entirely open radial domain ($0 \leq \rho < \infty$). Hence, the integral transform defined on the semi-infinite domain ($a \leq \rho < \infty$, $a \neq 0$), named the Weber transform, should be useful to deal with wave scattering problems defined on this region. Since the Weber transform can handle the semi-infinite domain, many important microwave device structures of circular apertures pierced by a cylinder can be analyzed by the Weber transform. While the Hankel transform has been extensively used in electromagnetic scattering and radiation problems, the Weber transform is relatively new in electromagnetic and microwave applications [23]. In what follows, we will briefly summarize some electromagnetic scattering and radiation problems that have been solved with the Weber transform.

3.1 Classical Weber Transform

Here, we consider an electrostatic problem of cylinder-penetrated circular aperture, as shown in Fig. 3(a). A long cylinder of radius a penetrates a circular aperture in a thick

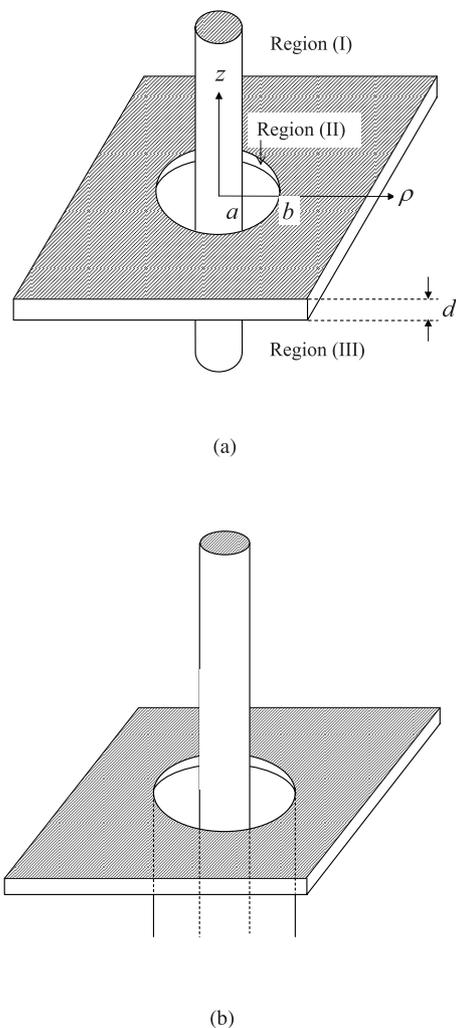


Fig. 3 (a) Cylinder-penetrated circular aperture in a thick conducting plane of infinite extent. (b) Infinitely long monopole antenna driven by a coaxial cable.

plane of thickness d and has its axis coincident with the z -axis. This is a fundamental yet important problem that deals with the field behavior of many practical structures (e.g., via holes or antenna feeds). We note that the electrostatic field behavior by an infinitely long cylinder penetrating a circular aperture was discussed in [24]. The cylinder is at a potential V_0 while the plane is kept at zero potential. The electrostatic potential Φ should satisfy Laplace's equation (1). Based on the superposition, the total electrostatic potential in region (I) is

$$\Phi^I = \Phi_a^I + \Phi_b^I \tag{9}$$

Here the primary potential Φ_a^I is represented in Fourier sine transform. Note that the primary potential is the contribution from V_0 at the cylindrical surface when the aperture is absent. We use the Weber transform to represent the secondary potential Φ_b^I . This transform pair is defined as [25]

$$\tilde{f}(\zeta) = \int_a^\infty f(\rho) Z_\nu(\zeta\rho) \rho d\rho \tag{10}$$

$$f(\rho) = \int_0^\infty \tilde{f}(\zeta) \frac{Z_\nu(\zeta\rho)}{J_\nu^2(\zeta a) + N_\nu^2(\zeta a)} \zeta d\zeta \tag{11}$$

where $Z_\nu(\zeta\rho) = J_\nu(\zeta\rho)N_\nu(\zeta a) - N_\nu(\zeta\rho)J_\nu(\zeta a)$, and $J_\nu(\cdot)$ and $N_\nu(\cdot)$ are the Bessel functions of the first and second kinds of order ν , respectively. Based on this Weber transform, Φ_b^I in (9) takes the form of

$$\Phi_b^I = \int_0^\infty \tilde{\Phi}^I(\zeta) e^{-\zeta z} \frac{Z_0(\zeta\rho)}{J_0^2(\zeta a) + N_0^2(\zeta a)} \zeta d\zeta \tag{12}$$

Similarly, by the superposition, the total potential in region (II) can be expressed as the sum of two potentials

$$\Phi^{II} = \Phi_a^{II} + \Phi_b^{II} \tag{13}$$

Note that Φ_a^{II} is written in series of the Bessel functions with unknown modal coefficient b_m , and can be seen as the secondary term to satisfy the boundary conditions. The potential Φ_b^{II} in (13) is a term that is attributed to the potential V_0 at the cylindrical surface. The boundary condition on the potential continuity at $z = 0$ is

$$\Phi^I|_{z=0} = \begin{cases} \Phi^{II}|_{z=0}, & a < \rho < b \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

Applying the Weber transform to (14), we obtain $\tilde{\Phi}^I(\zeta)$ in series of the modal coefficient b_m . The other boundary condition of the normal derivative continuity for $a < \rho < b$ is given by

$$\left[\frac{\partial \Phi^I}{\partial z} \right]_{z=0} = \left[\frac{\partial \Phi^{II}}{\partial z} \right]_{z=0} \tag{15}$$

The simultaneous equation of the modal coefficient b_m can be found by using the orthogonality relationship of the Bessel functions. Substituting $\tilde{\Phi}^I(\zeta)$ into (15) and applying the orthogonality, we obtain a set of simultaneous equations for the discrete modal coefficients b_m . To check the numerical efficiency, computation for equipotential contours is performed. Table 1 illustrates b_m up to $N = 15$ terms, indicating fast convergence of the series solution. Using the mode-matching method in conjunction with the Weber transform, we developed a full-wave solution to the problem of electromagnetic power transmission through a circular aperture pierced by a long cylinder [26]. The excitation was assumed to be a time-harmonic electric ring current source.

3.2 Other Type Weber Transforms

The classical Weber transform is limited by the requirement of the Dirichlet condition at the cylindrical surface $\rho = a$ although it is useful. There exist some other type Weber transforms that may be useful under special circumstances. To deal with the arbitrary mixed boundary condition, we should rely on the extended version of the classical Weber transform. This transform pair is called the generalized Weber transform and defined as [27]

$$\tilde{f}(\zeta) = \int_a^\infty f(\rho) D_\nu(\zeta\rho) \rho d\rho \tag{16}$$

Table 1 Convergence behavior of b_m for $b/a = 2.2$, $d/a = 0.6$ and $V_0 = 10$.

m	b_m
1	-11.496
2	3.134
3	-1.045
4	0.126
5	0.065
6	-0.063
7	0.040
8	-0.016
9	5.293×10^{-3}
10	-7.603×10^{-4}
11	-3.731×10^{-4}
12	4.076×10^{-4}
13	-2.589×10^{-4}
14	1.065×10^{-4}
15	-3.627×10^{-5}

$$f(\rho) = \int_0^\infty \tilde{f}(\zeta) \frac{D_\nu(\zeta\rho)}{Q_\nu^2(\zeta a)} \zeta d\zeta \quad (17)$$

where

$$D_\nu(\zeta\rho) = J_\nu(\zeta\rho) [\beta N_\nu(\zeta a) + \alpha \zeta N'_\nu(\zeta a)] - N_\nu(\zeta\rho) [\beta J_\nu(\zeta a) + \alpha \zeta J'_\nu(\zeta a)] \quad (18)$$

$$Q_\nu^2(\zeta a) = [\beta J_\nu(\zeta a) + \alpha \zeta J'_\nu(\zeta a)]^2 + [\beta N_\nu(\zeta a) + \alpha \zeta N'_\nu(\zeta a)]^2 \quad (19)$$

and the symbol ' denotes the differentiation with respect to the argument. By properly choosing two constants α and β , we can solve a certain class of boundary-value problems with arbitrary mixed conditions at the cylindrical surface. It is also apparent that the above generalized Weber transform becomes the classical Weber transform given by (10) and (11) for the case of $\alpha = 0$ and $\beta = 1$. The problem of acoustic power transmission into a cylinder-penetrated circular aperture was solved in [28] by using this generalized Weber transform. The boundary conditions are assumed to be acoustically hard (Neumann condition) and the values of constants used in the problem are $\alpha = 1$ and $\beta = 0$. Computations were performed to study the acoustic power transmission for various aperture geometries.

An important observation at this stage is that all of the previous transforms are comprised of the linear combinations of the Bessel functions of same order ν . However, for some applications in scattering theory, the aforementioned same-order Weber transforms are not sufficient to handle problems having boundary conditions of different types. In the general case we can generalize the Weber transform by introducing the Bessel functions of different orders as [29]

$$\tilde{f}(\zeta) = \int_a^\infty f(\rho) R_{\mu,\nu}(\zeta\rho) \rho d\rho \quad (20)$$

$$f(\rho) = \int_0^\infty \tilde{f}(\zeta) \frac{R_{\mu,\nu}(\zeta\rho)}{J_\nu^2(\zeta a) + N_\nu^2(\zeta a)} \zeta d\zeta \quad (21)$$

where $R_{\mu,\nu}(\zeta\rho) = J_\mu(\zeta\rho)N_\nu(\zeta a) - N_\mu(\zeta\rho)J_\nu(\zeta a)$. Note that (20) and (21) are usually called the associated Weber transform and for $\mu = \nu$ reduce to the classical Weber transform (10) and (11), respectively. By using this transform, we solved the problem of an infinitely long monopole antenna driven by a coaxial cable in [30], as shown in Fig. 3(b). In [31], the power transmission problem into a cylinder-penetrated circular aperture was also considered when the aperture structure was excited by a magnetic current loop. The mode-matching solutions are all shown to be efficient and numerically robust.

4. Conclusion

The mode-matching applications to the problems of circular and annular apertures in a thick perfectly conducting plane were reviewed. Basic electrostatic boundary-value problems were presented using the Hankel and Weber transforms. The electrostatic, magnetostatic, acoustic, and electromagnetic wave scattering problems were briefly presented. The mode-matching method enables us to obtain rigorous and numerically efficient solutions to many practically important problems encountered in circular and annular aperture scattering and radiation. The mode-matching approach can be further extended to other practical problems for circular and annular scattering in microwave and millimeter wave regimes.

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