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Diffraction-Free Bessel Beams at mm- and Submm-Wavebands

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SUMMARY Bessel beams are a family of diffraction-free beams. They have many unique properties and prospective applications. Much attention has been focused to this subject in optics. Recently, the studies of such beams at mm- and submm- wavebands have been carried out in our group. The investigation results, including their theories, generation, propagation and potential applications, are presented in this paper.

key words: *diffraction-free, Bessel beams, millimeter- and submillimeter-wavebands (mm- and submm-wavebands)*

1. Introduction

In 1987, the concept of "diffraction-free beams" was introduced into optics by Durnin for the first time [1]. This means that there is no change in the cross-section when such beam propagates in free space [2]. It is Bessel beams that are one and most interesting family of diffraction-free beams. Ideal Bessel beams possess many novel properties, such as large depth of field, propagation invariant and reconstruction [3], [4] and so on. Therefore, they have many significant applications, and increasing attention has been paid to this field. More recently, we have carried out the studies of Bessel beams in mm- and submm- range. The main purpose of this paper is to present our investigation results synthetically, covering their theories, generation, propagation and prospective applications [5].

The paper is organized as follows. The scalar and vector analyses of Bessel Beams are described in Sect. 2. Section 3 introduces how to generate approximate Bessel beams. The comparison of propagation between apertured Bessel and Gaussian beams is presented in Sect. 4. Many potential applications and conclusions are given in the last Sect. 5.

2. Scalar and Vector Analyses of Bessel Beams

As shown by Durnin [1], in the cylindrical coordinates system the free space wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial^2 t}\right)E(\rho, \varphi, z, t) = 0 \tag{1}$$

has the exact solution

$$E(\rho, \varphi, z, t) = E_0 J_n(k_\perp \rho)\exp(in\varphi)\exp[i(k_z z - \omega t)] \tag{2}$$

for a scalar field E propagating in a source-free region $z \geq 0$. Where E_0 is a constant, J_n is the nth-order Bessel function of the first kind, $k_\perp^2 + k_z^2 = k^2 = (2\pi/\lambda)^2$, $k_\perp$ and k_z are the radial and longitudinal wave numbers, respectively, λ is the free space wavelength, $\rho^2 = x^2 + y^2$. When k_z is real, the solution represents a diffraction-free field in the sense that the time-averaged intensity I is independent of z

$$I(\rho, \varphi, z \geq 0) = \frac{1}{2}|E(\rho, \varphi, z, t)|^2 = I(\rho, \varphi, z = 0) \tag{3}$$

When $n = 0$, Eq. (2) represents the zero-order Bessel beams (denoted by J_0 beams) discovered firstly by Durnin in 1987 [1]. The central spot of a J_0 beam is always bright, as shown in Figs. 1(a) and (b). The size of the central spot is determined by $k_\perp$, and when $k_\perp = k$, it reaches the minimum possible diameter of about $3\lambda/4$, but when $k_\perp = 0$, Eq. (2) reduces to a plane wave. The intensity profile of a J_0 beam decays at a rate proportional to $(k_\perp\rho)^{-1}$, so it is not square integrable [1]. However, its phase pattern is bright-dark interphase concentric fringes, as depicted in Fig. 1(c). But for $n > 0$, Eq. (2) denotes the high-order Bessel beams (i.e., J_n beams, n is an integer). The intensity distribution of all the higher-order Bessel beams has zero on axis surrounded by concentric rings. For example, when $n = 3$, the J_3 beam has a dark central spot and its first bright ring appears at $\rho = 4.201/k_\perp$, as illustrated in Figs. 2(a) and (b). However, the phase pattern of the J_n beam is much different from that of the J_0 beam. It has $2n$ arc sections distributed evenly from the innermost to the outermost ring, as plotted in Fig. 2(c).

In order to discover more characteristics of Bessel beams, the vector analyses should be performed. Many helpful research results have been presented in Refs. [6]–[8], and the vector analysis methods are usually based on auxiliary functions, such as vector potential [6], angular spectrum [7], and vector wave function [7], [8]. There are still many other important properties of Bessel beams that have not been developed but could be significant for future researches and applications. Therefore, we have also analyzed the vector Bessel beams based on the Hertz-vector potentials. And the resultant expressions for TM and TE modes Bessel beams can be given respectively by [9].

TM_n mode:

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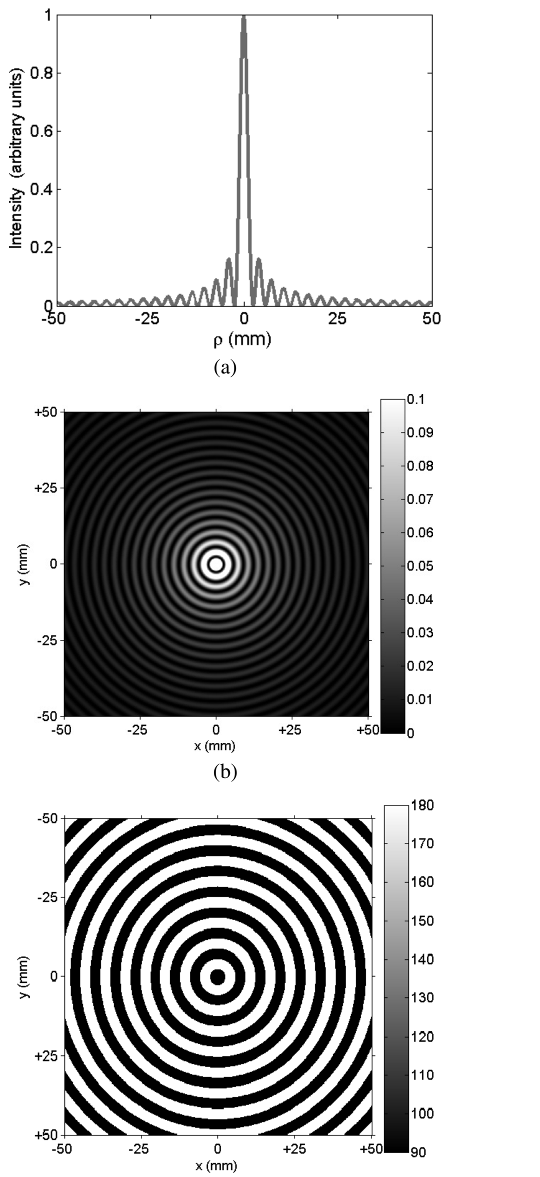

Fig. 1 A J_0 beam. (a) One-dimensional (1-D) intensity distribution. (b) 2-D intensity distribution plotted in a gray-level representation. (c) Phase distribution (t = 0, z = 0). The relevant parameters are incident wavelength of $\lambda = 3\,\text{mm}$, and aperture radius of $R = 50\,\text{mm}$, $k_\perp = 0.962\,\text{mm}^{-1}$.

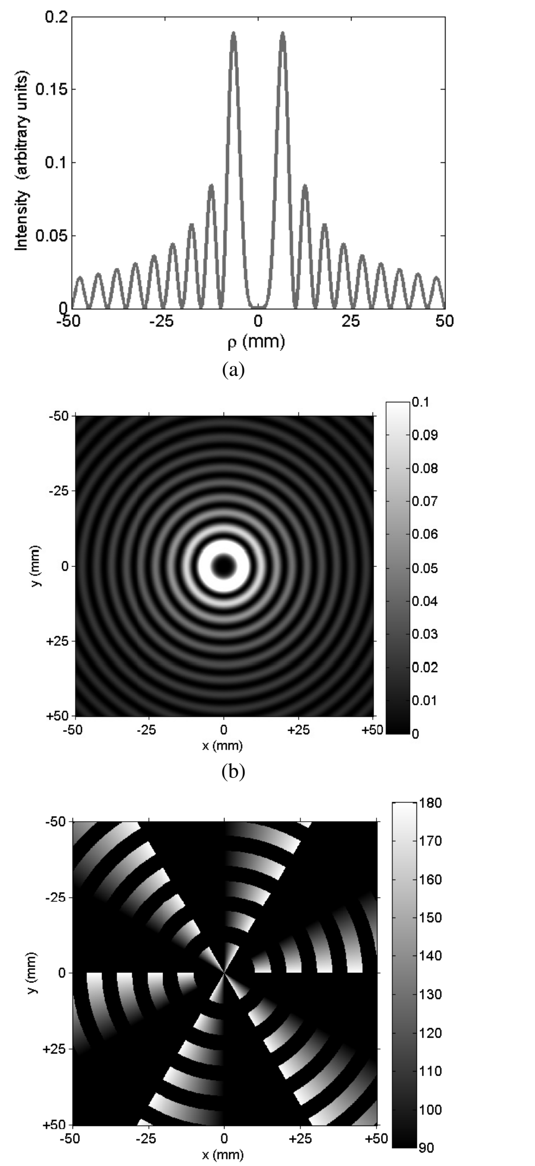

Fig. 2 A J_3 beam. (a) 1-D intensity distribution. (b) 2-D intensity distribution plotted in a gray-level representation. (c) Phase distribution (t = 0, z = 0). The relevant parameters are the same as in Fig. 1, except $k_\perp = 0.638\,\text{mm}^{-1}$.

$$\left.\begin{aligned}
E_{\rho e} &= iP_e k_\perp k_z J_n'(k_\perp\rho)\exp(in\varphi)\exp[i(k_z z-\omega t)]\\
E_{\phi e} &= -\frac{n}{\rho}P_e k_z J_n(k_\perp\rho)\exp(in\varphi)\exp[i(k_z z-\omega t)]\\
E_{ze} &= P_e k_\perp^2 J_n(k_\perp\rho)\exp(in\varphi)\exp[i(k_z z-\omega t)]\\
H_{\rho e} &= \frac{n}{\rho}P_e\omega\varepsilon J_n(k_\perp\rho)\exp(in\varphi)\exp[i(k_z z-\omega t)]\\
H_{\phi e} &= iP_e k_\perp\omega\varepsilon J_n'(k_\perp\rho)\exp(in\varphi)\exp[i(k_z z-\omega t)]\\
H_{ze} &= 0
\end{aligned}\right\} \quad (4)$$

TE_n mode:

$$\left.\begin{aligned}
E_{\rho m} &= -\frac{n}{\rho}P_m\omega\mu J_n(k_\perp\rho)\exp(in\varphi)\exp[i(k_z z-\omega t)]\\
E_{\phi m} &= -iP_m k_\perp\omega\mu J_n'(k_\perp\rho)\exp(in\varphi)\exp[i(k_z z-\omega t)]\\
E_{zm} &= 0\\
H_{\rho m} &= iP_m k_\perp k_z J_n'(k_\perp\rho)\exp(in\varphi)\exp[i(k_z z-\omega t)]\\
H_{\phi m} &= -\frac{n}{\rho}P_m k_z J_n(k_\perp\rho)\exp(in\varphi)\exp[i(k_z z-\omega t)]\\
H_{zm} &= P_m k_\perp^2 J_n(k_\perp\rho)\exp(in\varphi)\exp[i(k_z z-\omega t)]
\end{aligned}\right\} \quad (5)$$

where P_e and P_m are the electric and magnetic dipole moments, respectively; ω, ε and μ are the angular frequency, permittivity and permeability, respectively. From Eqs. (4)

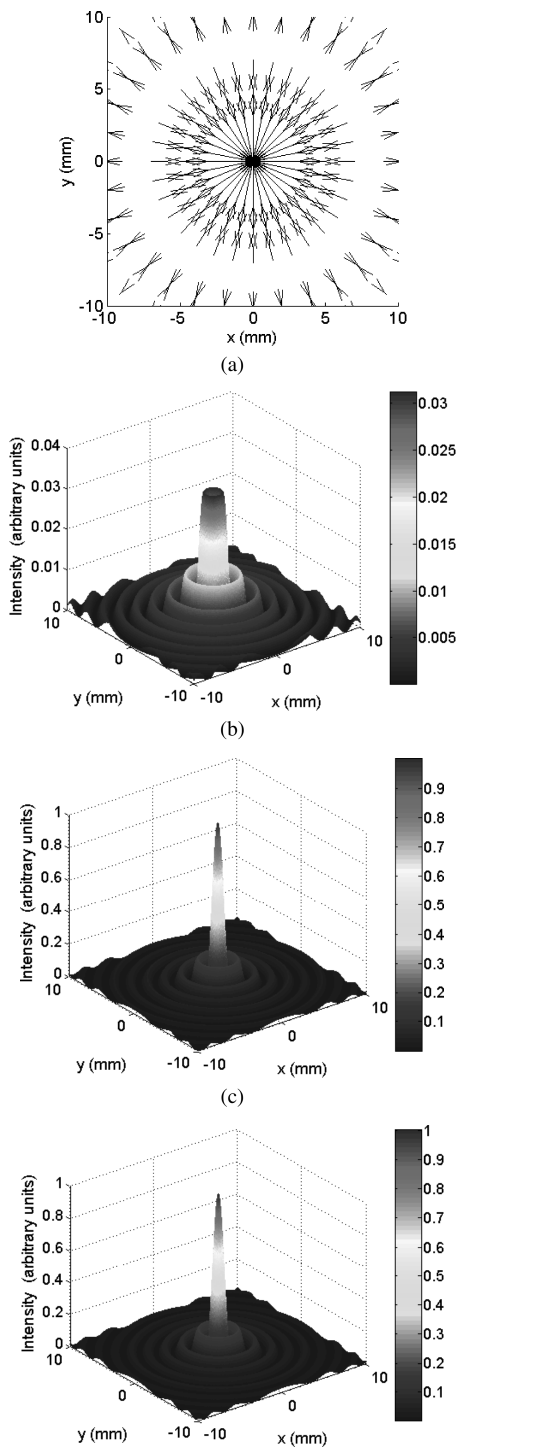

Fig. 3 TM_0 mode Bessel beam. (a) Instant vector diagram for the transverse component of the electric field ($t = 0$, $z = 0$). (b) The transverse electric field intensity ($I_\perp = |E_{\rho e}|^2 + |E_{\phi e}|^2$). (c) The longitudinal electric field intensity ($I_z = |E_{ze}|^2$) and (d) the total electric filed intensity ($I = I_\perp + I_z$). The color bars illustrate the relative intensity. The relevant parameters are $\lambda = 3$ mm, $k_\perp = 2.004\,\text{mm}^{-1}$, $k_z = 0.608\,\text{mm}^{-1}$, and $R = 10$ mm.

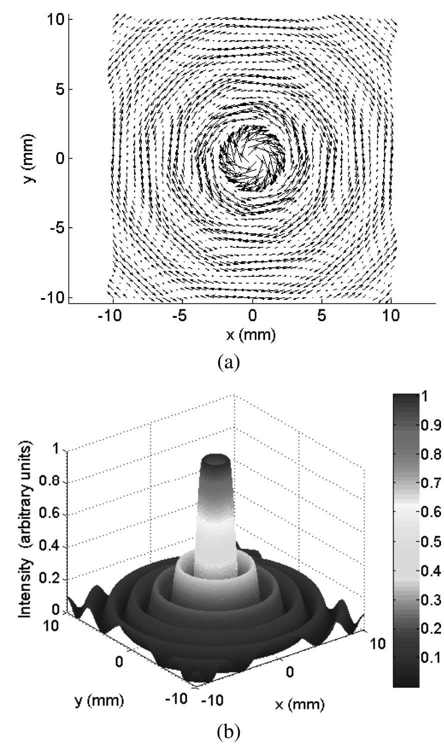

Fig. 4 TE_0 mode Bessel beam. (a) Instant vector diagram for the transverse component of the electric field ($t = 0$, $z = 0$). (b) The transverse electric field intensity. The parameters used here are the same as in Fig. 3, except $k_\perp = 1.503\,\text{mm}^{-1}$, and $k_z = 1.459\,\text{mm}^{-1}$.

and (5), the instant field vectors and intensity distributions for the TM or TE modes Bessel beams can be easily obtained. Two examples for TM_0 and TE_0 modes Bessel beams are illustrated in Figs. 3 and 4, respectively. From Eq. (4), we can see that the transverse electric field component of the TM_0 mode is only a radial part and thus it is radially polarized, as illustrated in Fig. 3(a). Similarly, the TE_0 mode is only an azimuthal component of the electric field and thus is azimuthally polarized. Its field vectors at $t = 0$ are shown in Fig. 4(a).

3. Production of Approximate Bessel Beams

An ideal Bessel beam extends infinitely in the radial direction and contains infinite energy, and therefore a physically generated Bessel beam is only an approximation to the ideal [10]. In optics, lots of methods for producing Bessel beams have been proposed, such as narrow annular slit [11], computer-generated holograms (CGH's) [12], Fabry-Perot cavity [13], axicon [14], optical refracting systems [15], diffractive phase elements (DPE's) [16] and so on. However, in the mm- and submm- wavelength range, relatively few methods for generating Bessel beams have been proposed currently. The method of using axicon [10] is very simple, but only a zero-order Bessel beam can be generated. The other method, using amplitude holograms [17], can produce arbitrary order Bessel beams, but only about 45% diffraction efficiencies can be reached. In order to overcome these limitations mentioned above, we have employed diffractive optical elements (DOE's), which have been designed in 2-D [18], [19] and 3-D form [20] respec-

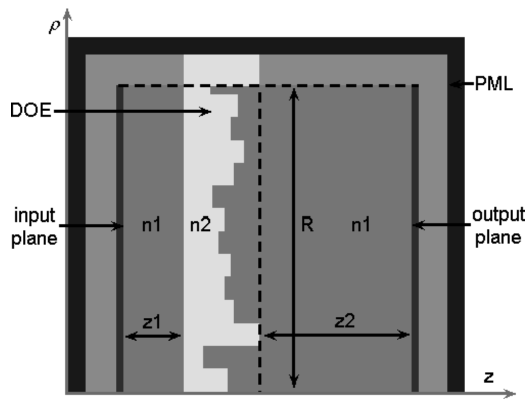

Fig. 5 Schematic diagram of computational model.

tively, to generate Bessel beams at mm- and submm- wavelengths for the first time. The computational model for our design is depicted schematically in Fig. 5, where the DOE with aperture radius R is utilized to convert an incident Gaussian beam on the input plane into a Bessel beam on the output plane. z_1 is the distance between the input plane and the DOE, and z_2 is the distance between the DOE and the output plane; n_1 and n_2 represent the refractive indices of the free space and the DOE, respectively. To determine the field diffracted by the DOE on the output plane, a two-dimensional finite-difference time-domain (2-D FDTD) and a body-of-revolution finite-difference time-domain (BOR-FDTD) method are used for 2-D and 3-D design, respectively. Apparently, the different DOE profile will generate the different field distribution. To find the DOE profile required to produce a Bessel beam on the output plane, a conventional genetic algorithm (GA) and microgenetic algorithm (MGA) are employed to perform these tasks in 2-D and 3-D design, respectively. Only parts of the 2-D [19] and 3-D [20] results are presented in Figs. 6 and 7, respectively. The Numerical results demonstrate that the designed DOE's can easily create arbitrary order Bessel beams when compared with axicons, and have higher diffraction efficiencies when compared with amplitude holograms.

4. Comparison of Propagation

The most interesting and attractive characteristic of such beam is diffraction-free propagation. In optics, the comparisons of maximum propagation distance had been done between apertured Bessel and Gaussian beams by Durnin [1] and Sprangle [21], respectively. However, their comparison results are conflict due to their different comparison criteria. Because Bessel beams have many potential applications at mm- and submm- wavebands, therefore, it is necessary and significant that the comparison is carried out at these bands. A new comparison criterion in the spectrum of mm- and submm-range has been proposed by us [22]. This criterion is the same initial total power and central peak intensity on the same initial aperture. In order to compare conveniently, we also defined the propagation distance as the

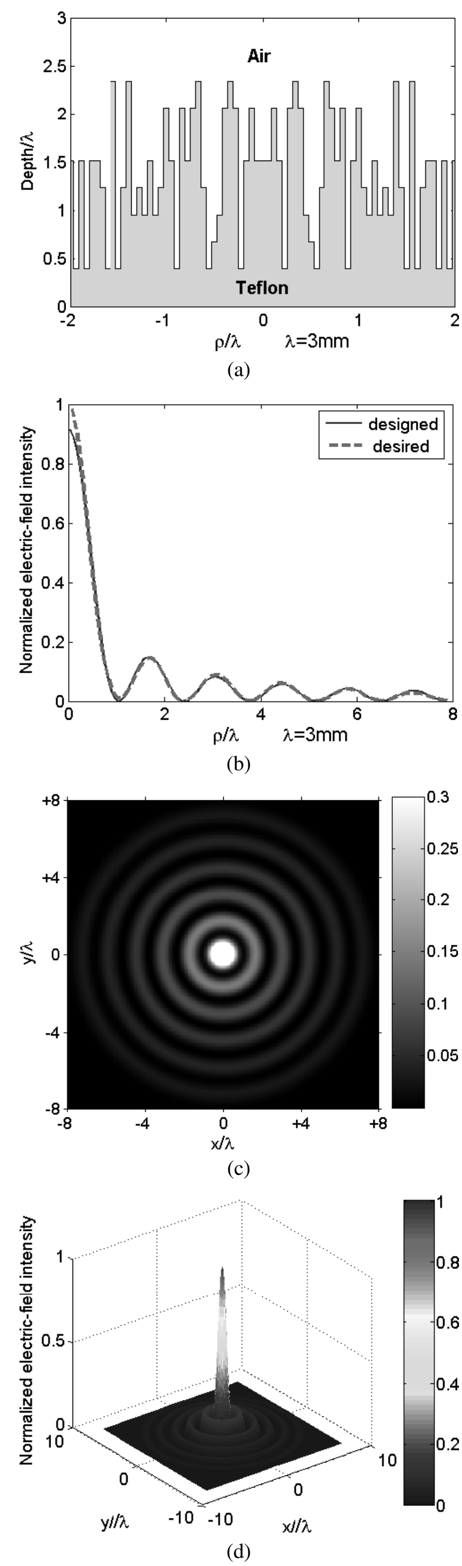

Fig. 6 Generation of a J_0 beam designed in 2-D form. $k_\perp = 0.7635\,\mathrm{mm}^{-1}$, the diffraction efficiency of $\eta = 94.494\%$. (a) Part of the optimized DOE profile. (b) The desired and the designed transverse intensity distribution on the output plane. (c) The 2-D transverse intensity distribution plotted in a gray-level representation, and (d) the 3-D transverse intensity distribution.

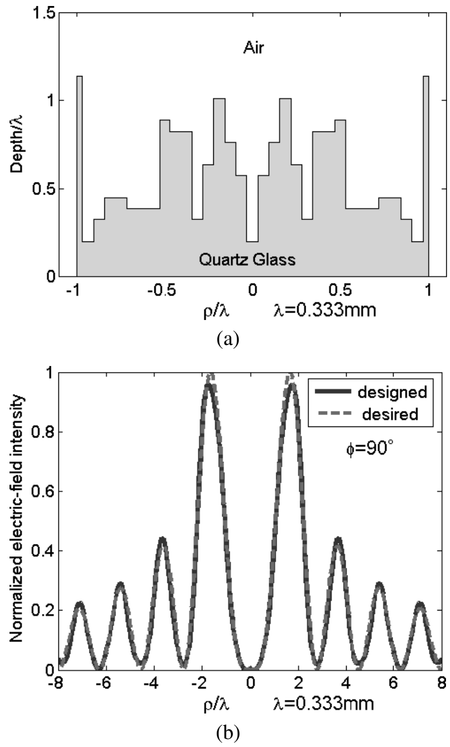

Fig. 7 Production of a J_2 beam designed in 3-D form. $k_\perp = 5.5538\,\mathrm{mm}^{-1}$ and $\eta = 96.232\%$. (a) Part of the optimized DOE profile, and (b) line scan of the desired and the designed electric field intensity distribution on the output plane at azimuthal angle of $\varphi = 90°$.

value of z-axis at which the axial intensity falls to 1/2. The comparison results, obtained by using Stratton-Chu formulas instead of Fresnel-Kirchhoff diffraction integral formula, are illustrated in Figs. 8 and 9. Under the proposed standard, a conclusion can be easily reach from Figs. 8 and 9 that the propagation distance of the apertured Bessel beam is greater than that of the apertured Gaussian beam. From Figs. 8(b) and 9(b), we can also observe that the axial intensity distributions of Bessel beams oscillate more acutely than those of Gaussian beams. This is because the initial field distributions of Bessel beams near the edges of the aperture are much larger that those of Gaussian beams, and as a result, Bessel beam will suffer more diffraction on the sharp edges of the aperture than Gaussian beams.

5. Applications and Conclusions

The novel properties of the diffraction-free Bessel beams have many significant applications [23]. In optics, due to the propagation invariance and extremely narrow intensity profile, Bessel beams are applicable in metrology for scanning optical systems. These beams are also suitable for large-scale straightness and measurements [24], since they can stand the atmospheric turbulence more than other beams. The imaging applications of the diffraction-free Bessel beams are also presented in [25], and it has been demonstrated that the imaging produced by Bessel beams can provide a longer focal depth when compared with Gaus-

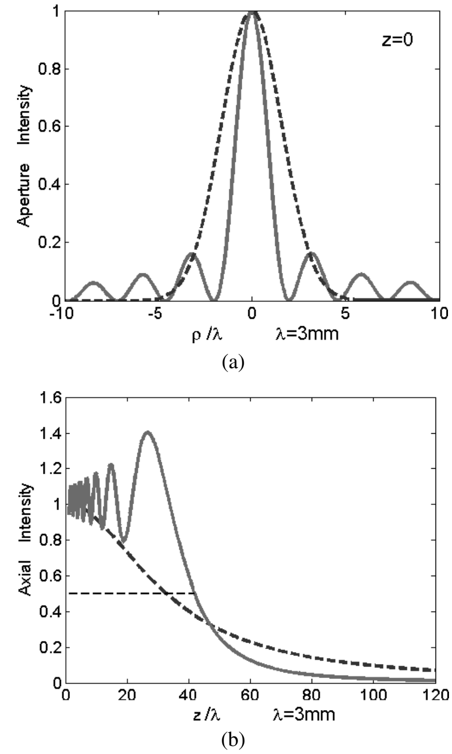

Fig. 8 The first case. (a) Intensity distributions for an apertured Bessel beam (—) and an apertured Gaussian beam (- - -) on the incident plane. (b) Axial intensities versus propagation distance z.

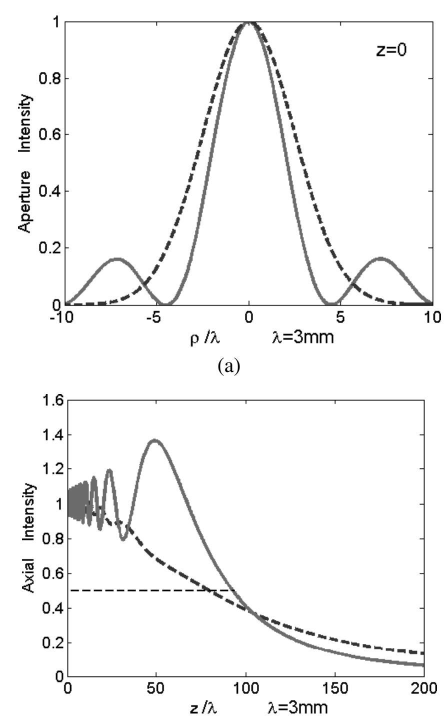

Fig. 9 The other example. (a) Initial Intensity distributions. (b) Their propagation distance.

sian beams. Bessel beams can also be applied to optical interconnection and promotion of free electron laser gain [26]. An increasing attention is devoted to the applications of Bessel beams in nonlinear optics. The third-harmonic generation using Bessel beams was proposed by Tewari and coworkers [27]. In addition, Cerenkov second-harmonic generation by nondiffracting Bessel beams in bulk optical crystals was also suggested in [28]. The application of Bessel beams to increase the Z-scan sensitivity in measurement was demonstrated in [29]. The radially polarized Bessel beams were applicable to accelerate the particles of the electron beam [30]. Recently, Bessel beams are used to manipulate micrometer-sized particles. Using the self-reconstruction property of Bessel beam, it is possible to manipulate tiny particles simultaneously in multiple planes [31], [32]. We believe that Bessel beams are prospective for improving the resolution of images in millimeter wave imaging system [10]. These beams may also be useful for measurements and power transmission at mm- and submm- wavebands.

In optical region of the spectrum, diffraction-free Bessel beams have attracted much interest over the years and have been widely investigated. However, in the mm- and submm- wave regions, exploratory work devoted to this field is much less. So, a great deal of contribution should be made to this field in the future.

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