1. Introduction

Uncertain data management is attracting considerable interest due to inherent uncertainty in real-world applications such as sensor-monitoring systems. For example, when analyzing purchasing behavior of customers using RFID sensors, there may be inaccurate sensor readings because of errors. In addition to this example, uncertain data take place in many situations. The major cause of uncertainty involves limitations of equipment, privacy concerns, or statistical methods such as forecasting. To deal with such uncertain data, "uncertain databases" have recently been developed [2].

In the area of uncertain data management, frequent itemset mining [3] from uncertain databases is one of the important research issues. Since the uncertainty is represented by probability, this problem is called "probabilistic frequent itemset mining." Many algorithms have been proposed to tackle probabilistic frequent itemset mining [6], [15], [17], [18]. However, existing algorithms suffer from performance problems because the computation of probability is highly time-consuming. It is thus necessary to accelerate this computation in order to handle large uncertain databases.

To this end, GPGPU (General-Purpose computing on Graphics Processing Unit) is an attractive solution. GPGPU refers to performing computation on GPUs (Graphics Processing Units), which are originally designed for processing 3D graphics. GPGPU has received much attention from not only the field of high performance computing but also many other fields such as data mining [5], [7], [12], [14], [16]. This is because GPUs have much more than hundred processing units and can process many data elements with high parallelism. It is also known that GPUs are energy-efficient and have higher performance-to-price ratios than CPUs. In addition, because the GPU architecture is relatively simpler than the CPU architecture, GPUs have been evolving rapidly [20].

By leveraging such a promising processor, our previous work [9] accelerated an algorithm of probabilistic frequent itemset mining. Meanwhile, GPU clusters, which are computer clusters where each node has one or more GPUs, have emerged as a powerful computing platform, such as HA-PACS," TSUBAME2.0," and Titan." Thus, it is increasingly important to harness the power of multiple GPUs and GPU clusters. The utilization of multiple GPUs gives us further parallelism and larger memory spaces. However, employing multiple GPUs has the problem that each GPU has a separate memory space. If one GPU requires data that reside in another GPU, the data need to be communicated via PCI-Express bus. Since the PCI-Express latency is much higher than the GPU-memory latency, it is probable that the communication becomes a bottleneck. It is therefore desirable to reduce data dependencies among data fragments on different GPUs.

This paper proposes multi-GPU methods that take into account the above concerns. First, we develop methods on a single node with multiple GPUs, and then we extend the methods to use a GPU cluster. The proposed methods reduce data dependencies by distributing candidates of probabilistic frequent itemsets among GPUs. In addition, the methods consider load balancing, which is also an important issue to achieve scalability. Experiments show that the single-node methods realize near-linear speedups, and the methods on a GPU cluster of eight nodes achieve up to a 7.1 times speedup.

The rest of this paper is organized as follows. Section 2 explains preliminary knowledge of proposed methods. Then Sect. 3 describes our proposed methods that utilize multiple GPUs. The methods are empirically evaluated in Sect. 4. Section 5 reviews related work, and Sect. 6 concludes this

SUMMARY Probabilistic frequent itemset mining, which discovers frequent itemsets from uncertain databases, has attracted much attention due to inherent uncertainty in the real world. Many algorithms have been proposed to tackle this problem, but their performance is not satisfactory because handling uncertainty incurs high processing cost. To accelerate such computation, we utilize GPUs (Graphics Processing Units). Our previous work accelerated an existing algorithm with a single GPU. In this paper, we extend the work to employ multiple GPUs. Proposed methods minimize the amount of data that need to be communicated among GPUs, and achieve load balancing as well. Based on the methods, we also present algorithms on a GPU cluster. Experiments show that the single-node methods realize near-linear speedups, and the methods on a GPU cluster of eight nodes achieve up to a 7.1 times speedup.

key words: GPU, uncertain databases, probabilistic frequent itemsets
2. Preliminaries

This section describes necessary knowledge to understand our proposed methods. Section 2.1 defines the problem that this paper deals with, i.e., probabilistic frequent itemset mining. Then a baseline algorithm [15] is explained in Sect. 2.2. The algorithm on a single GPU [9] is described in Sect. 2.3.

2.1 Probabilistic Frequent Itemsets

Let $I$ be a set of all items. A set of items $X \subseteq I$ is called an itemset, and a $k$-itemset means an itemset that contains $k$ items. An uncertain transaction database $\mathcal{U}$ is a set of transactions, each of which is a triplet of an ID, an itemset, and an existential probability. An existential probability stands for the probability that a transaction really exists in the database. Table 1 shows an example of uncertain transaction database, where each row corresponds to a transaction.

Uncertain transaction databases are often interpreted with possible worlds model. This model considers that an uncertain transaction database generates possible worlds. Each possible world is a distinct subset of the database and represents an instance of the database. For example, the database of Table 1 generates 16 possible worlds: $\emptyset, \{T_1\}, \{T_2\}, \{T_3\}, \{T_4\}, \{T_1, T_2\}$, etc. Each possible world is probabilistically generated according to the existential probabilities. For instance, the world $\{T_1, T_2\}$ is generated with the probability that $T_1$ and $T_2$ exist, and $T_3$ and $T_4$ do not exist. Hence the probability is $0.6 \cdot 0.8 \cdot (1 - 0.5) \cdot (1 - 0.3) = 0.168$.

The concept of support is used to define conventional frequent itemsets. The support of an itemset $X$ is denoted as $\text{sup}(X)$ and means the number of transactions that include $X$. An itemset $X$ is called a probabilistic frequent itemset (PFI) if

$$P(\text{sup}(X) \geq \text{minsup}) = \sum_{i=\text{minsup}}^{\text{mnsup}} f_X(i) \geq \text{minprob}, \quad (1)$$

where $\mathcal{U}$ is an uncertain transaction database, and $\text{mnsup}$ and $\text{minprob} \in (0, 1]$ are user-specified thresholds. Probabilistic frequent itemset mining is to extract all probabilistic frequent itemsets, given an uncertain transaction database, $\text{minsup}$, and $\text{minprob}$.

2.2 pApriori Algorithm

Sun et al. [15] proposed a pApriori algorithm, which adapts the classical Apriori algorithm [4] to uncertain databases. The pApriori algorithm comprises two procedures:

1. Generating a set of candidate $k$-itemsets $C_k$ from a set of $(k - 1)$-PFIs $L_{k-1}$
2. Extracting a set of $k$-PFIs $L_k$ from $C_k$

The pApriori algorithm continues these procedures alternately with incrementing $k$ by one until no additional PFIs are detected. Note that, in the beginning of the algorithm, $k$’s value is one and candidate 1-itemsets are singletons that contain items in an input database $\mathcal{U}$. Eventually the pApriori algorithm returns all the PFIs extracted from $\mathcal{U}$.

2.2.1 Generating Candidates

Candidates are generated in two phases: merging and pruning phases.

The merging phase checks whether pairs of $(k - 1)$-PFIs are joinable: Two $(k - 1)$-itemsets $X$ and $Y$ satisfy the condition $X \cdot \item_{1} = Y \cdot \item_{1} \land \cdots \land X \cdot \item_{k-2} = Y \cdot \item_{k-2} \land X \cdot \item_{k-1} < Y \cdot \item_{k-1}$, where $X \cdot \item_{i}$ denotes the $i$th item of $X$. If $X$ and $Y$ are joinable, a new candidate $k$-itemset is created as the union of $X$ and $Y$. This candidate is stored to a set of candidate $k$-itemsets $C_k$.

The pruning phase prunes candidates using the following lemma [15]:

**Lemma 1** (Anti-monotonicity). If an itemset $X$ is a PFI, then any itemset $X' \subset X$ is also a PFI.

The contraposition of this lemma yields that if any itemset $X' \subset X$ is not a PFI, then the itemset $X$ is not a PFI either. Hence a candidate $k$-itemset can be pruned out when any size-$(k - 1)$ subset of the candidate is not contained in a set of $(k - 1)$-PFIs $L_{k-1}$. If the candidate can be pruned out, it is deleted from $C_k$.

2.2.2 Extracting Probabilistic Frequent Itemsets

In order to determine whether or not an itemset $X$ is a PFI, the SPMF of $X$ needs to be computed and assigned to
Eq. (1). However, a naive solution to compute SPMFs is considered to be intractable, because the number of possible worlds is exponentially large. To address this problem, Sun et al. [15] proposed two algorithms: dynamic-programming and divide-and-conquer approaches. They showed that the divide-and-conquer algorithm is more efficient.

Although the algorithm is able to compute SPMFs in \(O(n \log n)\) time, it is still a time-consuming task. Thus, it is desirable to prune infrequent itemsets without computing the SPMFs. Let \(\text{cnt}(X)\) be the number of transactions that include an itemset \(X\) in an uncertain transaction database \(\mathcal{U}\) regardless of existential probabilities. Besides, let \(\text{esup}(X)\) be the expected value of \(\text{sup}(X)\). With the two values, Sun et al. proved two lemmas that enable us to prune candidates in \(O(n)\) time [15].

### 2.3 Single-GPU Parallelization

The single-GPU method [9] follows the pApriori algorithm and consists of generating candidates and extracting PFIs. Candidates are generated on a GPU by a parallel version of the algorithm described in Sect. 2.2.1. Then PFIs are extracted with four steps: inclusion check, pruning, filtering, and computing SPMFs. For more details, readers are encouraged to refer to the paper [9].

Inclusion check judges whether transactions include each candidate. Since the information that a transaction includes a candidate can be represented by one bit, the result of inclusion check for a candidate is stored as an array of bitstrings, called an inclist. By using inclists of \((k-1)\)-itemsets, inclusion check for \(k > 1\) can be processed fast. The inclist of a \(k\)-itemset \(X\) is computed as bitwise AND operations between two inclists of \((k-1)\)-PFIs that are used to create the itemset \(X\).

In the pruning step, candidates are pruned by computing the two values mentioned in Sect. 2.2.2. Then, non-pruned candidates become subject to the filtering step. For each candidate, this step filters out transactions that do not include the candidate, because such transactions do not contribute to the SPMF of the candidate. With this filtering, the number of transactions to compute the SPMF of an itemset \(X\) decreases from \(|\mathcal{U}|\) to \(\text{cnt}(X)\). The next step computes the SPMF of this candidate and the algorithm determines whether the candidate is a PFI or not.

### 3. Multi-GPU Parallelization

In a multi-GPU system, each GPU has a separate memory space. If data dependencies exist among GPUs, GPUs need to communicate with each other via PCI-Express bus. Thus it is important how to distribute data to be processed on GPUs, so that data dependencies are minimized.

Multi-GPU systems can be considered as a kind of distributed-memory systems. Meanwhile, there exists much work on frequent itemset mining for distributed-memory systems. Zaki [19] classified data-distribution schemes that existing algorithms employ into three: count distribution, data distribution, and candidate distribution. In this paper, we propose methods employing the candidate-distribution scheme, because this scheme enables GPUs to compute SPMFs independently, unlike the other two schemes.

Section 3.1 describes methods on a single node equipped with multiple GPUs. Then, Sect. 3.2 extends the methods to use a GPU cluster.

### 3.1 Single-Node Methods

#### 3.1.1 Prefix Distribution

We here describe an algorithm based on a naive candidate-distribution scheme. This algorithm is called Prefix Distribution (PD), because the algorithm distributes candidates by exploiting the property that two candidates with different prefixes are not joinable. Therefore if we partition candidates with prefixes, GPUs can generate candidates using their own PFIs. Algorithm 1 shows a pseudocode of PD. PD is divided into three phases: initialization, distribution, and loop phases.

In the initialization phase, PD extracts 1-PFIs. PD firstly generates candidate 1-itemsets from an input uncertain transaction database, and then conducts inclusion check and pruning on the CPU in Line 2. By processing inclusion check and pruning on the CPU rather than on GPUs, the amount of data transfer to GPUs reduces, because the database is unnecessary in later iterations. Although inclusion check and pruning are performed individually in the single-GPU method, because it is more suitable for GPU’s architecture, these operations can be executed simultaneously on the CPU. Thus PD performs these two operations at the same time. Non-pruned candidates are evenly distributes to GPUs (Line 3). Inclists of all the non-pruned candidates, which are necessary in later inclusion check, and the array of probabilities are also transferred to GPUs. Then GPUs carry out filtering and compute SPMFs, and copy 1-PFIs to the CPU (Lines 4–5).

In the distribution phase, PD generates candidate 2-itemsets from the 1-PFIs on the CPU in Line 7, and dis-

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**Algorithm 1: Prefix distribution**

1. **Initialization** \((k = 1)\)
2. Inclusion check and pruning on the CPU
3. Distribute candidates \(C_1\) and transfer inclists to GPUs
4. Filtering and computing SPMFs on GPUs
5. Gather 1-PFIs from GPUs to the CPU
6. **Distribution** \((k = 2)\)
7. Generate candidate 2-itemsets on the CPU
8. Distribute the candidates \(C_2\) to GPUs by prefix
9. **while a set of candidates \(C_k \neq \emptyset\)**
   10. // Inclusion check uses inclists of \((k-1)\)-itemsets
11. Extract \(k\)-PFIs on GPUs
12. Gather the \(k\)-PFIs from GPUs to the CPU
13. Broadcast all the \(k\)-PFIs to GPUs
14. \(k \leftarrow k + 1\)
15. Generate candidate \(k\)-itemsets on GPUs
Candidates are balanced. In a round-robin fashion, instead of using prefixes. In the distribution phase, candidates are allocated to GPUs.

Although data transfer between CPU and GPUs occurs at each iteration, only candidates are transferred and the communication time is negligible in practice.

The second modification is for inclusion check to employ inclists of 1-itemsets rather than \((k-1)\)-itemsets. In the case of PD, inclusion check of a \(k\)-itemset \(X\) can be performed with inclists of \((k-1)\)-itemsets that are origins of \(X\), because \(X\) and the origins reside in the same GPU. On the other hand, since RRD distributes candidates in round robin, it is probable that the two origins and their inclists reside in different GPUs. Hence, if a GPU tries to perform inclusion check with inclists of \((k-1)\)-itemsets, the GPU may need to access inclists in another GPU. Consequently data transfer between GPUs, which should be avoided, occurs. To address this issue, we utilize the inclists of 1-itemsets that were already transferred in the initialization phase. In this case, inclusion check of a \(k\)-itemset is performed as \(k-1\) bitwise AND operations among inclists of \(k\) 1-itemsets included in \(X\).

### 3.1.3 Count-Based Distribution

We describe another candidate-distribution scheme called Count-Based Distribution (CBD). CBD assigns candidates to GPUs by taking into account candidates’ cnt values. The rationale is that the cnt values determine the computing time of SPMFs\([15]\), and the computing time of SPMFs dominates the processing time of candidates on GPUs. Therefore we can achieve load balancing if candidates are distributed by considering cnt values. To this end, we firstly need to estimate the computing times of SPMFs for particular cnt values.

We can estimate the computing time of an SPMF for a particular cnt value, if we know the computing time of an SPMF for the next-higher power of two of the cnt value. This is because the single-GPU method uses \(2^{\lceil \log_2 \text{cnt}(X) \rceil}\) transactions to compute the SPMF of an itemset \(X\) in order to simplify the computation on a GPU. In this work, we measure in advance the computing times of SPMFs for power-of-two numbers of transactions. Once the computing times are measured, this information can be used for any dataset, because the computing times are determined only by execution environments and are independent on datasets.

To achieve load balancing, we also need to know the cnt values of candidates when assigning candidates to GPUs. If \(k = 1\), the values are computed before the assignment. On the other hand, if \(k > 1\), the cnt values are not computed at the assigning time, and thus need to be estimated. We here approximate \(\text{cnt}(X)\) as \(\min_{X' \subset X, |X'| = k-1} \text{cnt}(X')\). This value is an upper bound of the actual \(\text{cnt}(X)\) value, and \(\text{cnt}(X)\) is expected to be less than this upper bound. Although the difference between the upper bound and \(\text{cnt}(X)\) might become large, the difference is negligible in practice, because estimating a computing time requires only the next-higher power of two of \(\text{cnt}(X)\).

The algorithm of CBD is identical to RRD except for its candidate-distribution scheme. CBD distributes candidates to GPUs so that processing times on GPUs are as even as possible. The algorithm of candidate distribution firstly

![Fig. 1](prefix_distribution.png)

(a) Candidates are imbalanced. (b) Candidates are balanced.
Table 2  Characteristics of datasets.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Type</th>
<th>Number of items</th>
<th>Avg. size of transactions</th>
<th>Number of transactions</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accidents</td>
<td>real</td>
<td>468</td>
<td>33.8</td>
<td>340,183</td>
<td>7.2%</td>
</tr>
<tr>
<td>T25I10D500K</td>
<td>synthetic</td>
<td>7558</td>
<td>25</td>
<td>499,960</td>
<td>0.33%</td>
</tr>
<tr>
<td>Kosarak</td>
<td>real</td>
<td>41270</td>
<td>8.1</td>
<td>990,002</td>
<td>0.020%</td>
</tr>
</tbody>
</table>

Fig. 2  Speedup of multi-GPU methods compared to the single GPU method on a single node.

initializes candidates and a processing time of each GPU to empty set and zero, respectively. Then the algorithm iterates the following two steps over candidates:

1. Find the GPU that has the minimum processing time.
2. Update candidates and processing time of the GPU.

Having finished the distribution, CBD extracts PFIs on GPUs as in RRD.

3.2 A Method on a GPU Cluster

This section describes a method on a GPU cluster that is an extension of single-node methods described in Sect. 3.1. For the sake of simplicity, this paper assumes that all the nodes hold an input database.

The basic idea is identical to the single-node methods: Candidates are distributed among nodes and are processed in parallel. The detailed algorithm is as follows. First, each node extracts PFIs from candidates, which are evenly partitioned among nodes. Each node processes distinct candidates, and thus generates distinct PFIs. To generate next candidates, the PFIs extracted on a node need to be broadcasted to all other nodes.

Subsequently, candidate 2-itemsets are generated. Either RRD or CBD is used to determine which node processes specific candidates. The same candidate-distribution scheme is used to further distribute the candidates among GPUs within a node. Then, each node extracts 2-PFIs, and broadcasts the PFIs to other nodes. The above steps continue with incrementing $k$ by one until there is no candidate. Since only PFIs need to be communicated among nodes, the communication cost is very small and negligible.

4. Experiments

4.1 Experimental Setup

We implemented the proposed methods using CUDA [20], OpenMP, and MPI. Experiments were conducted on a GPU cluster of eight nodes, each of which has two CPUs and two GPUs. The CPU is Intel Xeon E5645 with 6 cores running at 2.4 GHz. The GPU is NVIDIA Tesla M2090 with 512 cores running at 1.3 GHz. The nodes are connected via InfiniBand QDR (Quad Data Rate).

Table 2 summarizes the three datasets used in the experiments. The density of a dataset is computed as the average length of transactions divided by the number of items. Accidents and Kosarak are real datasets that are accessible on Frequent Itemset Mining Implementations (FIMI) Repository. While Accidents is the densest dataset, Kosarak is the sparsest dataset. T25I10D500K is a synthetic dataset, generated by a data generator.

The default values of $\text{minsup}$ on Accidents, T25I10D500K, and Kosarak are $33\%$, $0.65\%$, and $0.2\%$, respectively. Existential probabilities for the datasets are randomly drawn from a normal distribution with mean 0.5 and variance 0.02. The value of $\text{minprob}$ is fixed to 0.5 for all the experiments.

4.2 Results on a Single Node

This section evaluates the three methods on a single node (PD, RRD, and CBD). We first show the result of whole algorithms and then analyze the several aspects of the algorithms in detail. Figures 2(a)–2(c) show the speedups of the three methods compared to the single-GPU method [9], with varying $\text{minsup}$ values. We measured execution time as elapsed time from when the dataset is ready on CPU to when all the result are collected to CPU. Note that the communication time between CPU and GPU is included in the execution time. From the charts, we can make the following three observations:

1. The multi-GPU methods become increasingly faster.

http://fimi.cs.helsinki.fi/

http://miles.cnuce.cnr.it/~palmeri/datam/DCI/datasets.php
than the single-GPU method, as the minsup value decreases.
2. RRD and CBD are generally faster than PD.
3. CBD outperforms RRD on Kosarak, while CBD and RRD exhibit similar performance on Accidents and T25I10D500K.

The first observation is due to the fact that small minsup values make the number of candidates large, and thus GPUs have much more workload to be done in parallel. Although the number of candidates is small on Accidents, Fig. 2(a) shows near-linear speedups. This is because the cnt values become very large (nearly 250,000) on Accidents, and hence the computation of SPMFs on Accidents is far more computationally demanding than the computation on T25I10D500K and Kosarak. As a result, the overall processing time on GPUs is dominated by the time of computing SPMFs, which can be performed in parallel. Note that the RRD and CBD on Kosarak achieve super-linear speedups as shown in Fig. 2(c). This is because candidate generation on a CPU is faster than that on GPUs if the number of generated candidates becomes very large.

The second observation results from the load imbalance in PD. To verify this observation, we use a load imbalance factor defined as the ratio of the longest processing time of GPUs to the shortest processing time of GPUs. The factor takes the value of 1 if the load is completely balanced, and the factor increases as the load is more imbalanced. Figures 3(a)–3(c) show the load imbalance factors between two GPUs. These charts reveal that RRD and CBD achieve better load balancing than PD, and realizes higher speedup ratios, as shown in Figs. 2(c) and 3(c). On the other hand, CBD and RRD on Accidents and T25I10D500K exhibit similar performance as shown in Figs. 2(a) and 2(b). This is because the cnt values (i.e., processing times) of candidates in Accidents and T25I10D500K do not vary much. Thus it is sufficient to distribute candidates in round robin in order to achieve load balancing.

As a summary of this section, we demonstrate how fast our methods run compared to the serial execution on CPU. Figures 4(a)–4(c) shows the speedup values. On Accidents, RRD and CBD achieve the speedup of about 350; on t25i10d500k, they are 90 times faster. The results on these two datasets exhibit stable speedup with respect to minsup values. On the other hand, the speedup on Kosarak decreases as making the minsup value smaller. This is because when a minsup value is small, the number of candidates with small cnt values increases and more time is spent for the filtering step.

4.2.1 Time Breakdown at Each Iteration

This section further analyzes the most efficient method, CBD, to clarify influences of each step of the algorithm on overall execution time. Figures 5(a)–5(c) show the execution time of five steps at each iteration. The five steps consists of candidate generation, inclusion check, pruning, filtering, and computing SPMFs. The time of pruning at k = 1 is not shown because it is contained in inclusion check.

The most influential step is computing SPMFs on all the three datasets. On Accidents, this step takes 90% of overall execution time. When using T25I10D500K and Kosarak, CBD spends about 60% of overall time. On these datasets, more time is spent at inclusion check and pruning compared to the case of Accidents. This is due to the larger number of candidates and the larger sizes of datasets.

4.2.2 Larger Memory Footprint

In this section, we make memory footprint on GPU larger by lowering minsup values on Kosarak, to see the advantage of larger memory space with multiple GPUs. Figure 6
describes the execution time of single-GPU method, RRD, and CBD. Some points of the single-GPU method are not shown because it does not work due to running out of memory. Table 3 shows the sizes of inclists, which are the largest data on GPUs.

The single-GPU method works only when the \( \text{minsup} \) value is greater or equal to 0.18%. This result and Table 3 indicate that the single-GPU method can deal with inclists of 3.6 GB but cannot deal with inclists of 4.1 GB. On the other hand, RRD and CBD work with the cases of smaller \( \text{minsup} \) values. Since RRD and CBD use two GPUs, they can handle inclists of 7.0 GB (\( \text{minsup} = 0.14 \)), which is about 2 times of 3.6 GB. However, RRD and CBD are also running out of memory when \( \text{minsup} = 0.13 \). In this case, the size of inclists is 8.6 GB, which is more than two times of 4.1 GB.

### 4.2.3 CPU–GPU Communication Time

This section evaluates the transferred data size and communication time between CPU and GPU. Figures 7(a) and 7(b) show the sizes of transferred data between CPU and GPU on Accidents and Kosarak, respectively. Itemset in the figures include candidates and PFIs. We show only the result of CBD because other methods show similar results. In addition, we omit the result on T25I10D500K because it is similar to the result of Kosarak.

On Accidents, the array of probabilities is the largest transferred data and itemsets are the smallest data. Inclists lie in the middle of these two data. On the other hand, inclists become the largest transferred data on Kosarak. This is because the number of 1-PFIs on Kosarak, which corresponds to the number of inclists, is much larger than the number of 1-PFIs on Accidents.

Figures 8(a) and 8(b) illustrate communication time between CPU and GPU on Accidents and Kosarak, respectively. In general, it can be seen from the figures that the communication time is proportional to the data size. One exception is Itemset on Accidents; the communication takes almost the same time as inclists, although itemsets are small. This is because itemsets are communicated multiple times,
4.3 Results on the GPU Cluster

This section evaluates two variants of the method on the GPU cluster with two candidate-distribution schemes, namely RRD and CBD. Figures 9(a)–9(c) show execution times of RRD and CBD and their breakdowns with varying number of nodes and the default values of minsup. Load imbalance factors among all GPUs are also shown in Figs. 10(a)–10(c). We measured execution time as elapsed time between when all nodes finish loading data and when a master node collects all PFIs. The execution time consists of three parts: communication time among nodes (Comm), execution time on GPU (GPU), and execution time on CPU (CPU). More specifically, GPU comprises inclusion check for $k > 1$, pruning for $k > 1$, filtering, and computing SPMFs. CPU includes inclusion check and pruning for $k = 1$, candidate generation, and data transfer between CPU and GPU. As mentioned in Sect. 3.2, we assume that all the nodes have the complete dataset. Thus the dataset does not need to be communicated among the nodes.

On Accidents, Figs. 9(a) and 10(a) show that CBD achieves slightly better load balancing than RRD and a 7.1 times speedup on eight nodes. Since the computation of SPMFs on Accidents is computationally expensive as mentioned in Sect. 4.2, GPUs can have much work processed in parallel. Thus, CBD and RRD show high scalability.

Figure 9(b) shows that CBD and RRD result in the almost same performance on T25I10D500K, and the speedup on eight nodes is 4.5. The methods on this dataset do not achieve high scalability as in the case of Accidents. This is because cnt values on T25I10D500K are small, and there is little work processed by GPUs. Thus the execution time on CPU, which is incurred by all nodes, dominates 10–25% of overall execution time, and becomes a bottleneck. Mean-
Table 4  Communicated data size among nodes.

(a) Accidents

<table>
<thead>
<tr>
<th>minsup (%)</th>
<th>31</th>
<th>32</th>
<th>33</th>
<th>34</th>
<th>35</th>
<th>36</th>
<th>37</th>
<th>38</th>
<th>39</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transferred data size (KB)</td>
<td>7.8</td>
<td>5.8</td>
<td>4.2</td>
<td>3.0</td>
<td>2.2</td>
<td>1.8</td>
<td>1.3</td>
<td>1.0</td>
<td>0.69</td>
</tr>
</tbody>
</table>

(b) T25I10D500K

<table>
<thead>
<tr>
<th>minsup (%)</th>
<th>0.65</th>
<th>0.675</th>
<th>0.7</th>
<th>0.725</th>
<th>0.75</th>
<th>0.775</th>
<th>0.8</th>
<th>0.825</th>
<th>0.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transferred data size (KB)</td>
<td>5.4</td>
<td>2.8</td>
<td>1.5</td>
<td>0.99</td>
<td>0.53</td>
<td>0.24</td>
<td>0.13</td>
<td>0.097</td>
<td>0.068</td>
</tr>
</tbody>
</table>

(c) Kosarak

<table>
<thead>
<tr>
<th>minsup (%)</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
<th>0.35</th>
<th>0.4</th>
<th>0.45</th>
<th>0.5</th>
<th>0.55</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transferred data size (KB)</td>
<td>9.8</td>
<td>6.1</td>
<td>4.2</td>
<td>2.9</td>
<td>2.1</td>
<td>1.7</td>
<td>1.4</td>
<td>1.1</td>
<td>0.94</td>
</tr>
</tbody>
</table>

while, the fact that CBD and RRD have similar performance is due to the little variability of cnt values on T25I10D500K, as discussed in Sect. 4.2.

On Kosarak, CBD achieves much better load balancing than RRD (Fig. 10(c)), and shows a 4.9 times speedup on eight nodes. Since candidates have a wide range of cnt values on this dataset, RRD cannot balance the load among nodes, while CBD accommodates to the variability. On average, CBD is 1.3 times faster than RRD on this dataset.

Figure 11 focuses on the communication time among nodes. Table 4 summarizes the sizes of communicated data among nodes. Since the proposed methods only communicate PFIs, their sizes are very small and not so different among datasets (Table 4). However, the communication time can vary largely. For example, the communication on Accidents takes 10 times longer than the communication on T25I10D500K (Figs. 11(a) and 11(b)). This is because the communication time include the synchronization time. In other words, each node needs to wait until all the nodes finish to find PFIs. That is why the communication takes much longer on Accidents, where the computation of SPMFs is very expensive. For the same reason, on average, the communication of CBD is shorter than that of RRD.

5. Related Work

This section reviews related work on frequent itemset mining and probabilistic frequent itemset mining.

Frequent itemset mining. The problem of association rule mining was firstly introduced by Agrawal et al. [3]. Association rule mining consists of two steps: frequent itemset mining and finding association rules. Since frequent itemset mining is more computationally intensive, many algorithms have been proposed to accelerate the mining. Among the algorithms, there are two major algorithms, namely Apriori [4] and FP-growth [8].

Parallelization of these algorithms has been widely studied. In the late 1990s, distributed-memory systems were mainly used as the underlying architecture. Zaki summarized such algorithms [19]. More recently, Li et al. [10] designed a parallel algorithm of FP-growth on a massive computing environment using MapReduce. Özkur et al. [13] introduced a data distribution scheme based on a graph-theoretic approach.

Frequent itemset mining on GPUs has been also studied [5], [7], [14], [16]. Fang et al. [7] proposed two approaches, GPU-based and CPU-GPU hybrid methods. The GPU-based method utilizes bitstrings and bit operations for fast support counting, running entirely on a GPU. The hybrid method adopts the trie structure on a CPU and counts supports on a GPU. Teodoran et al. [16] parallelized the Tree Projection algorithm [1] on a GPU as well as a multicore CPU. Amossen et al. [5] presented a novel data layout BatMap to represent bitstrings. This layout is well suited to parallel processing and compact even for sparse data. Then they make use of BatMap to accelerate frequent itemset mining. Silvestri et al. [14] proposed a GPU version of a state-of-the-art algorithm DCI [11].

Probabilistic frequent itemset mining. While the above-mentioned parallel algorithms work well for the con-
convventional certain transaction databases, they cannot effectively process frequent itemset mining from uncertain databases, which gains increasing importance in order to handle data uncertainty. There is much work for modeling, querying, and mining such uncertain data (see a survey by Aggarwal and Yu [2] for details and more information).

To mine frequent itemsets with taking into account the uncertainty, a number of algorithms have been proposed. Bernecker et al. [6] proposed an algorithm to find probabilistic frequent itemsets under the attribute-uncertainty model, where existential probabilities are associated with items. On the other hand, Sun et al. [15] considered the tuple-uncertainty model, where existential probabilities are associated with transactions. Recently, Tong et al. [17] implemented existing representative algorithms and test their performance with uniform measures fairly.

Several attempts to accelerate these algorithms also exist. Wang et al. [18] developed an algorithm to approximate the probability that determines whether itemsets are PFIs or not. Our previous work [9] presented an algorithm using a single GPU based on the work by Sun et al. [15]. We have extended this method to use multiple GPUs in this paper.

6. Conclusions

This paper has proposed methods of probabilistic frequent itemset mining using multiple GPUs. The methods run in parallel by distributing candidates among GPUs. The proposed method PD assigns candidates to GPUs according to their prefixes. Then we have described RRD and CBD, which take into consideration load balancing. RRD distributes candidates to GPUs in a round-robin fashion, while CBD uses the cnt values of candidates to achieve better load balancing. We have also presented methods on a GPU cluster by extending RRD and CBD. Experiments on a single node showed that CBD achieves the best load balancing and results in the fastest algorithm. Experiments on a GPU cluster of eight nodes revealed that our proposed methods are effective if there is sufficient workload to be processed on GPUs. Future work involves developing a CPU-GPU hybrid method that utilizes CPUs and GPUs simultaneously. In addition, more sophisticated multi-node methods should be considered.

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