Theoretical Study on Wave Propagation and Scattering in Random Media and Its Application

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1. Introduction

Theoretical study on wave propagation and scattering in random media has a long history and is said to originate in analysis of the twinkle of the stars. Multiple scattering in random media was investigated in the 1960s–1970s mainly in USA and USSR. The driving force behind the investigation was based on the solution of practical and core subjects for evaluating effects of random media on communications and remote sensing. The investigation is deeply related to physics and mathematics and has been carried out also from academic points of view [1]–[4]. Now the theoretical study has been worldwide promoted from different angles.

Deciding on the final goal that the theoretical results are used in various regions of science and technology and solving subjects in order to accomplish the goal, the author has been studying random medium problems from the beginning of 1970. The author’s study corresponds to engineering including basis and application in Table 1, and has been directed to the development of creative methods and applications.

As a result of the study, the following four subjects are described in this paper: (1) derivation and analysis of moment equations, (2) scattering by many particles and the effective medium constant of random medium, (3) scattering by a conducting body in random media and (4) spatially partially-coherent wave scattering. The paper space is limited so that main points of research on the above subjects would be shortly described and detailed results and data should be obtained from references.

2. Derivation and Analysis of Moment Equations

Suppose that the dielectric constant of random medium \( \varepsilon \) is a random function of time and space, the time change is much slower in the medium than the wave velocity, and the ergodic hypothesis is valid. Then \( \varepsilon \) may be treated as a function of space; and Maxwell’s equations for electromagnetic waves and Helmholtz’s equation for scalar waves, respectively, are given as follows.

\[
\begin{align*}
\nabla \times E(r, \omega) &= -j\omega\mu_0 H(r, \omega) \\
\nabla \times H(r, \omega) &= j\omega\varepsilon_0 E(r, \omega) \\
\n\nabla^2 + k^2(r) \mu(r, \omega) &= 0
\end{align*}
\]

where the wave number \( k(r) = \omega \sqrt{\varepsilon(r)\mu_0} \), the magnetic permeability \( \mu = \mu_0 \) (constant), and the time factor \( \exp(j\omega t) \) is suppressed.

It is difficult that we express solutions of (1) and (2) explicitly in a compact form by using known functions because \( \varepsilon(r) \) is a random function. In case effects of the random medium are weak, we can apply approximate methods useful for analyzing waves in inhomogeneous media: e.g. geometrical optics approximation and Rytov approximation in asymptotic analysis, and Born approximation in perturbation analysis. According to these methods, the amplitude or phase of wave depends linearly on \( \varepsilon(r) \), and the multiple scattering of wave by \( \varepsilon(r) \) is not sufficiently considered. Therefore analytic methods applicable also to the case that effects of the random medium become strong were studied mainly in USA and USSR during times from 1960s to 1970s. As a result, the moments required in practical applications, which are statistics of electromagnetic waves: e.g. \( \langle E(1) \rangle \), \( \langle E(1)E^*(2) \rangle \) and \( \langle |E(1)|^2|E(2)|^2 \rangle \), have been analyzed, where the number in parentheses indicates the position in space and occasionally the frequency, and the asterisk denotes complex conjugate.

The above analysis paid attention to the moments is called as dishonest method [1] by which practically useful results have been obtained up to now. In general, any order moments are given as

\[
M_p(q, \mu) = \left( \prod_{m=0}^{\mu} E(m)(p_m) \right) \left( \prod_{n=0}^{\nu} E^*_{jn}(q_n) \right)
\]

where \( E(m) \), \( E^*_{jn} \) are the vector components of \( E \), and \( p_m, q_n \) denote the space position and the frequency occasionally.
Moment equations that (3) satisfies were derived for successively forward-scattered waves on the assumption that $\epsilon(r)$ is Gaussian random and the correlation length $l$ is much larger than the wave length $\lambda$ in free space: $kl \gg 1$, $k = 2\pi/\lambda$. The deviation was done from different approaches by assuming in addition that $\epsilon(r)$ obeys Markov process in the wave-propagation direction, except for the author. A phase-changing screen model for random medium is physically intelligible to the layman [5]. However my approach based only on $kl \gg 1$ does not need the additional assumption and permits the inhomogeneous randomness of $\epsilon(r)$ in the wave-propagation direction [6]–[9], which result is useful for practical applications such as satellite communications. Analyzing the moment equations, the author has presented the following results: the mechanism of spot dancing of wave beams [10], reciprocity of electromagnetic waves in random media [11], [12] and analytic expressions of second moments [13]–[16]. These results have been applied to an evaluation of antenna gains [17], restoration of images degraded by random media [18], [19] and the BER analysis in Ka band satellite communications [20]. Much more still remains to be investigated in practical application.

3. Scattering by Many Particles and the Effective Medium Constant of Random Medium

Consider the scalar wave scattering problem shown in Fig. 1 where $\epsilon(r)$ in (2) is given by $\epsilon(r) = \epsilon_0\left(1 + \Delta \epsilon(r)\right)$ and $\Delta \epsilon(r)$ indicates the distribution of relative dielectric constants of particles. If we assume the wave in $V$ as the incident wave: i.e. Born approximation, then the assumption corresponds to the first term approximation $T(r'', r') = k^2\Delta \epsilon(r'')\delta(r'' - r')$ in Fig. 1. The solution of this problem is to obtain $T(r'', r')$; however, it is not easy for any distribution of $\Delta \epsilon(r)$.

Scattering by many particles has been analyzed by different methods according as the distribution of particles is periodic or random. To the periodic case Floquet’s theorem applies and the analysis results in scattering by only one particle; on the other hand, to the random case, so-called multiple scattering methods have been applied. Although periodic and random distributions are quite distinct from each other, the discrimination of the distributions becomes difficult when using waves of much longer wavelength than the average distance between particles. The author presented a method, called DUR method, applicable to the transition case from periodic to random distribution for scalar waves [21] and electromagnetic waves [22]. Figure 2 shows conventional and my approaches where the particle distributions are described. Using DUR method equations were approximately derived for the coherent wave $\langle E(r) \rangle$ and the second moment $\langle E(r_1)E'(r_2) \rangle$, and expressed in terms of two parameters of periodicity and randomness in the particle distribution.

By separating coherent Green’s function $(G(r))$ into the periodicity and randomness parts where well-known periodic Green’s function remains for non-random case and the periodicity in $\langle G(r) \rangle$ vanishes for strongly random case, the author investigated the condition for the distribution of particles to be random, which means that the periodicity part can be neglected in $(G(r))$. As a result, the conditions for dielectric spheres and cylinders were given by using the radius and relative dielectric constant of particles, the average distance between particles, and the frequency [23], [24].

According as the distribution of particles, effective dielectric constants of media $\langle \epsilon(r) \rangle$ are depicted as Fig. 3. The author evaluated the $\langle \epsilon(r) \rangle$ of media composed of dielectric spheres for the weakly and strongly random distributions: (b) and (c) in Fig. 3 and also did the $\langle \epsilon(r) \rangle$ for the case of dielectric cylinders [25], [26]. The $\langle \epsilon(r) \rangle$ of random media
has been evaluated by multiple scattering methods such as EFA: effective field approximation, QCA: quasi crystalline approximation and QCA-CP: QCA with Coherent Potential. As a result, the evaluation by DUR method is the best up to now and is applicable to random media composed of highly dielectric particles such as water drops for microwaves. This prominent result was applied to the detection problem of the water content in soil [27], [28] and also to the estimation of the effective medium parameters of random media composed of chiral particles [29]. The reference [30] shows the difference in multiple scattering among EFA, QCA, QCA-CP and DUR method. It is not easy to analyze the effective medium parameters if random medium consists of several kinds of particles. Therefore the analysis has been done numerically. Because the numerical analysis causes a large-scale calculation problem, a fast computation method adequate to support the time, memory and accuracy has been required; accordingly, we have studied the method and obtained some useful results [31], [32]. The study still continues on scattering by many particles.

### 4. Scattering by a Conducting Body in Random Media

To develop the radar technology, scattering by a target in random media should be analyzed as a boundary value problem. The author proposed a method to do that to the scattering problem shown in Fig. 4 [33], [34]. The outline of the method is given in Fig. 5, where the case of random medium is compared with the case of free space and the current generator plays an important role for the formulation. The generator, whose name is unfamiliar, depends only on the body, is valid for any incident wave and can be obtained by Yasuura’s method.

Using the proposed method, average scattering cross-sections have been calculated and shown to have characteristics peculiar to the case of random medium [35]–[39]. However, the vector analysis of the three dimensional problems remains untouched because of complexity. Through
Fig. 5  A schematic diagram for solving the scattering problem where $U_{in}$ is incident wave, $J$ current, $U_s$ scattered wave, and RM indicates random medium.

(a) in free space

(b) in random medium

Fig. 6  Two issues in the scattering problem.

issue 1: scattering for the incidence of spatially partially-coherent (SPC) wave

issue 2: statistical coupling of incident and scattered SPC waves through random medium

Fig. 7  Scattering from a body surrounded by a phase changing screen.

Fig. 8  A prototype of circular SAR.

the calculation it became clear [40], [41] that this scattering problem consists of two physical issues: that is, scattering by spatially partially-coherent (SPC) incident waves and statistical coupling of incident and scattered SPC waves through random medium, as shown in Fig. 6. Well-known backscattering enhancement occurs by issue 2 in Fig. 6; however, in practice such as radar engineering, we need the quantitative estimate of scattering cross-sections by taking account of issue 1. This fact is closely related to the subject of the next chapter.

5. Spatially Partially-Coherent Wave Scattering

Scattering by a body in free space has been basically studied on the assumption that the incident wave is perfectly-coherent in space. On the basic assumption many research results have been obtained and applied to practical cases; currently, the study of the scattering continues using large-scale computation techniques. If the incident wave is partially-coherent in space: i.e. SPC wave, then there is some possibility that the scattering cross-section of a body in free space differs much from that for the incidence of perfectly-coherent wave because large effects of the issue 1 described in Sect. 4 are partly known to the case of random medium. Therefore we need to investigate the scattering for the SPC wave incidence as a new subject. In order to do it, we consider the scattering problem shown in Fig. 7 where a conducting body is surrounded by a phase changing screen with infinitesimal thin layer. When changing randomly the phase, we can obtain the scattering cross-section similar to that for the case of random medium shown in Fig. 4 [42], [43]. This study is connected with the development of a new SAR system using SPC waves, as shown in Fig. 8. Scattering from a body illuminated by SPC waves is a challenging subject.

6. Concluding Remarks

The theoretical studies conducted mainly by the author are reviewed on wave propagation and scattering in random media and its application: in particular, on (1) derivation of arbitrary order moment equations and solutions of some equations, (2) scattering by many particles and the effective medium constant of random medium, (3) scattering by a conducting body in random media and (4) spatially
partially-coherent wave scattering, with application to satellite communications, artificial material development, and sensing and radar technology. The leading and/or outstanding points of the above researches are merely described and detailed results and data are referred to representative articles already published.

Acknowledgements

These researches have been conducted in cooperation with staffs including students and with the financial support of Japanese government and companies. I am thankful to Dr. N. Nakashima of Kyushu University for his help in this writing.

References


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